Volume VIII. *May–June, 1899.* *Number 5.*

THE

PHYSICAL REVIEW.

THE CONDUCTIVITY AND DISSOCIATION OF SOME ELECTROLYTES.[1]

By William Foster, Jr.

SOME time ago Loomis[2] determined the depression of the freezing-points of a number of dilute aqueous solutions, and for some of these no data for calculating the theoretical depression were at hand. To supply the necessary data this work was primarily undertaken, and at the same time occasion has been taken to measure the conductivities of some other electrolytes which have been more or less investigated by other observers.

The series of measurements comprises the following groups of compounds:

I. *Neutral Salts.*—Magnesium Sulphate, Magnesium Chloride and Ammonium Nitrate.

II. *Acid Salts.*—Sodium Ammonium Hydrogen Phosphate, Disodium Hydrogen Phosphate and Potassium Dihydrogen Phosphate.

III. *Weak Mineral Acid.*—Phosphoric.

IV. *Organic Acids.*—Oxalic and Citric.

V. *Strong Base.*—Sodium Hydroxide.

The Method and Apparatus.

The Kohlrausch method of the alternating current was employed. The Hartmann and Braun Roller Bridge was used, and the bridge-wire was carefully calibrated[3] when necessary by comparison with a set of standard resistances. The bridge-wire, which had been used for some time, changed but little in the course of several months.

[1] Part I. of Thesis presented to the Faculty of Princeton University for the degree of Doctor of Philosophy.

[2] Phys. Review, Vol. III., No. 16, 1896, and Vol. IV., No. 22, 1897.

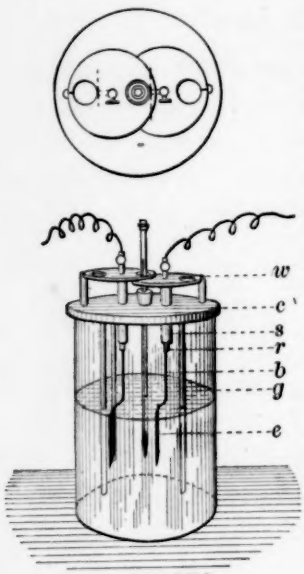
[3] Kohlrausch & Holborn, Leitvermögen der Elektrolyte. Leipzig, 1898, p. 48.

THE RESISTANCE VESSELS.

For the measurement of the resistances of the concentrated solutions, the vessel was of the ordinary double type.[1] (It had a "resistance capacity" of 1575.10^{-6}, referred to normal KCl as 918.10^{-8}.) The vessel and contents were kept in a water bath at a temperature of 18° C. for some time before the measurements were made.

THE LOOMIS RESISTANCE VESSEL.

For the measurement of the resistances of the more dilute solutions, a vessel of a different type was of course employed. This vessel was designed by Professor Loomis and was made in the Princeton Physical Laboratory. A strong, well-shaped beaker 14 cm. in height and 10 cm. in diameter was chosen. The rim was ground flat with emery, and a well-fitting cover of ebonite was provided. It is best described by reference to the diagram, Figs. 1 and 2, which show the vessel as a whole and also the plan of the cover and discs.



Figs. 1. and 2.
The Loomis Resistance Vessel.
e. Electrode, 4 × 3 cm.
b. Platinum Bar.
r. Brass Rod.
s. Brass Sleeve.
c. Ebonite Cover.
w. Brass Disc.
g. Glass Rod.

The platinum electrodes *e* (Fig. 1) are supported by rigid platinum bars *b*, bent at such an angle that the plane of each electrode is about 1 cm. out of the line of the supporting bar *b*. The platinum bars terminate in brass rods *r*, carefully fitted so as to turn in brass "sleeves" *s*, which are firmly secured in the ebonite cover *c* at a distance of 3.5 cm. apart.

This arrangement permits the electrodes to be turned around so their distance apart may be varied. The three electrode-positions which I have employed in the present work are the following, which will be denoted by *A*, *B* and *C* respectively:

[1] Kohlrausch, Leitfaden der Prak. Phys. Leipzig, 1896, 4th figure, p. 336.

In the *A*-position the electrodes are turned so to be the nearest possible to each other. Their distance apart is then 11 mm.

In the *B*-position the left-hand electrode is turned around 180°. A medium distance of 30 mm. is thus obtained.

In the *C*-position the other electrode is also turned 180°. A maximum electrode distance of about 53 mm. is thus obtained.

Fig. 2a.

To make sure of the greatest exactness in the constancy of the electrode positions, the sleeves in which the rods of the electrodes turn were made long (about 5 cm.), and each of the brass rods which turn in them carries at its upper end a brass disc on which are two fine marks that serve as index marks by which the discs may be set accurately in the desired positions.

The various dimensions and the non-essential parts are described in a footnote,[1] and data in regard to the constancy of the cell are given on page 260.

The Resistance Capacity of the Vessels.

The resistance capacity γ of the Loomis vessel was determined by use of a solution of potassium chloride $m = 0.01$ and 0.02. The conductivity of these solutions at 18° C. according to the most recent determinations of Kohlrausch[2] is 1146 and 2242 multiplied by 10^{-10} (Hg = 1). The following values for γ were obtained:

[1] The platinum electrodes are 4 x 3 cm. They were well platinized by the use of Lummer and Kurlbaum's solution (see Kohlrausch, Leitvermögen der Elektrol yte, p. 10)

The platinum bars are about 5 cm. long and 2 mm. in diameter.

The brass rods are about 6 cm. in length.

The brass "sleeves" are about 5 cm. in length and 1 cm. in diameter.

They are firmly fixed in the cover and are about 1.5 cm. from its center.

The discs are 5.5 cm. in diameter.

They are provided with two holes each 1.5 cm. in diameter.

The thermometer is thrust through these holes.

Four glass rods firmly fixed to the cover support the electrodes when they are lifted from the cell.

[2] Kohlrausch, Holborn & Diesselhorst, Wied. Ann., 64, 417, 1898.—"Neuer Grundlagen für die Werte der Leitvermögen der Elektrolyten." The values given there are reduced to the *old basis*.

POSITION A OF ELECTRODES.

Amount of Solution Used.	Gram. Equiv. of KCl per Liter.	Value of $\gamma \cdot 10^8$.	Mean Value of γ 10^8.
500 cc.	0.01	529.7	529.7
	0.02	529.6	
490	0.01	534.3	534.5
	0.02	534.7	
	POSITION B.		
500	0.01	938.9	939.0
	0.02	939.0	
490	0.01	954.6	954.4
	0.02	954.2	
	POSITION C.		
500	0.01	1296	1296
	0.02	1296	
490	0.01	1324	1324
	0.02	1324	

The values of γ obtained a month later are as follows:

	POSITION A.		
500	0.01	528.8	529.4
	0.02	529.6	
490	0.01	534.1	534.3
	0.02	534.5	
	POSITION B.		
500	0.01	938.0	938.8
	0.02	939.5	
490	0.01	954.3	953.9
	0.02	953.5	
	POSITION C.		
500	0.01	1297	1296.5
	0.02	1296	
490	0.01	1324	1323.5
	0.02	1323	

It thus appears that the cell is constant far within the limits of experimental error.

For the smaller vessel, a solution of KCl $m = 1$ and 0.1, and a solution of NaCl $m = 1$ were employed. The values of γ are given below:

	Gram. Equiv. of Salt per Liter.	Value of $\gamma \cdot 10^6$.	Mean Value of $\gamma \cdot 10^6$.
KCl	1.0	157.5	157.5
	0.1	157.6	
NaCl	1.0	157.5	

THE BATH AND TEMPERATURE AND GRADUATION OF APPARATUS.

A water bath was used for all of the measurements. The temperature was 18° C. The thermometer was graduated in tenths of a degree and was compared with a standard instrument from the Reichsanstalt. The flasks, pipettes, weights, thermometers, etc., were carefully standardized, and the weighings reduced to vacuum.

DETERMINATION OF SPECIFIC GRAVITY.

The sp. gr. of the normal solutions was determined. The method employed is that of Sprengel, somewhat modified.[1] Water at 4° C. is taken as unity. The volume of the pyknometer at 18° C. was 37.0642 true c. c.

THE UNITS EMPLOYED.

The resistances of the solutions were determined in the Siemens unit, and the specific conductivities are, therefore, referred to this unit, that is, the *unit* resistance was that of a column of mercury 100 cm. long and with a cross-section of 1 mm^2. The early results were generally expressed in this unit. Kohlrausch and others[2] now express the electrical conductivity of solutions in another unit, and for convenience of comparison with both the earlier and the later data, the measurements contained in this paper will be given in both units. According to the new system, the unit of conductivity is the conductivity of a body a column of which 1 cm. long, and 1 cm.^{2} in cross-section has a resistance of one ohm.

To place the old values of the equivalent-conductivity upon the *new basis*, one simply multiplies by the factor 0.1069, or

$$\Lambda = 0.1069 . 10^6 \frac{K}{m}.$$

To reduce the new to the old, divide by 0.1069.

[1] Loomis, PHYS. REVIEW, Vol. III., p. 270, 1896.

[2] Wied. Ann., 64, 417, 1898; and Leitvermögen der Eletrolyte, Leipzig, 1898.

ON THE PREPARATION OF PURE WATER.

The method of Dr. Hulett was[1] employed throughout the entire work. The distilled water ordinarily used in the Chemical Laboratory is first treated with a solution of potassium bichromate and sulphuric acid (about 10 liters of water to 50 c. c. each of $K_2Cr_2O_7$ and H_2SO_4). These act as a strong oxidizing agent. After standing some time, the water is distilled, treated with a saturated solution of barium hydroxide, and distilled again. From 10–20 c. c. of $Ba(OH)_2$ were used per liter. The essential feature of the Hulett method is that the condensing tube (platinum) is thrust into the neck of the retort so as to reach into the body of the retort itself. Thus, only the steam that condenses *inside* of the platinum tube finds its way into the collecting flask.

The retorts were kept under a hood in order that the CO_2 from the burning gas might not contaminate the atmosphere of the room. This work was begun in the spring, when the room was well aired almost daily. By rejecting the first portions of the distillate, and without taking any special precautions to keep out the CO_2, it was possible to prepare water of a high degree of purity. On one occasion 500 c. c. of this water were transferred to the large resistance vessel and K. 10^{10} was found to be as low as 0.78 ; and in a number of instances it was in the vicinity of 1. This was all that could be desired. Upon resuming the work, however, late in the fall it was observed that the water as prepared above had a greater conductivity. The conclusion was soon reached that the air of the room contained more CO_2 than it did in the spring, so a soda-lime tube was connected with the receiving flask. This had the desired effect, and it was no longer difficult to prepare water of the desired purity. The conductivity of the water was substracted from that of the solution of the electrolyte. Experiments made with water of different degrees of purity (from K.10^{10} = 2.5 to 1) justified this course.

THE CONCENTRATIONS OF THE SOLUTIONS.

Measurements were made in general on solutions ranging from a gram-equivalent, or normal, solution to $m = 0.0001$. The normal

[1] Journal of Phys. Chem., Sept., 1896.

solution was first prepared, and from this the concentrations up to $m = 0.03$ were prepared by direct dilutions.

For the more dilute solutions the following plan[1] was pursued: 500 c. c. were put into the Loomis resistance vessel and its conductivity determined, then 9.5 c. c. of the water were taken out, and by means of a 0.5 c. c. pipette (exact capacity 0.4994 c. c.) and a 2 c. c. pipette (exact capacity 2.0004 c. c.) 0.5, 0.5, 2 and 2 c. c. of the solution $m = 0.10$ were successively added, the resistance being determined after each addition; next, 0.5, 2 and 2 c. c. of the solution $m = 1$ were added. The pipettes were drained about 10 seconds after the addition of the solution and then blown out three times. Experiments made with the pipettes showed this to be more exact than could be obtained by simple draining.

By following this plan, solutions were prepared of the following concentrations:

$m = 0.0001017,\ 0.0002032,\ 0.0006077,\ 0.001009,\ 0.002015,$
$0.006024,\ 0.01000.$

Solutions in which $m = 0.01$ were made up directly as a control to the indirect concentration of the same value, and the two values are given side by side in the tables below.

The Preparation of the Compounds.

Due attention was given to the preparation of the compounds. Recrystallization and gravimetric and volumetric analyses were frequently resorted to. The sp. gr. of the normal solutions was also determined. Table I. below comprises the principal facts regarding each compound.

Column 1 gives the molecular formula; 2, the concentration in *gram-equivalents* per liter; 3, the grams of the compound per liter at 18° C.; 4, the sp. gr. $\frac{18°}{4°}$; 5, the electrical conductivity; 6, source of material, and 7, how the solution was prepared.

[1] Kohlrausch, Wied. Ann., 26, 1885, page 175.

TABLE I.

1 Compound.	2 m.	3 Grams per Liter at 18°C.	4 Sp. Gr. $\frac{18°}{4°}$ C.	5 Elec. Cond. K. 10^6.	6 Source of Material.	7 How Prepared.
KCl (Standard)	1	74.59	1.0449	918	Eimer & Amend.	By direct Wt.
NaCl (Standard)	1	58.50	1.0392	695[1]	" " "	" " "
NH_4NO_3	1	80.10	1.0310	830.	Made in Lab.	" " "
$MgCl_2$	1	47.60	1.03815	593.5	Eimer & Amend.	By analysis.
$MgSO_4$	1	60.18	1.0572	270.	Trommsdorff.	" "
$NaNH_4HPO_4$	1	45.71	1.0345	283.	Merck & Co.	By Wt.
Na_2HPO_4	1	47.38	1.0429	262.	Eimer & Amend.	" "
KH_2PO_4	1	45.39	1.0301	200.	" " "	" "
H_3PO_4	1	32.68	1.0165	208.	Trommsdorff.	By analysis.
$(COOH)_2$, $2H_2O$	1	63.01	1.0203	556.	Merck & Co.	" "
$C_6H_8O_7$, H_2O	1	70.02	1.0254	50.	Kahlbaum.	By titration.
NaOH	1	40.06	1.0420	1469.	Made in Lab.	" "

THE STANDARD POTASSIUM CHLORIDE.

The ordinary c. p. KCl was recrystallized, and chemical tests proved it to be of a high degree of purity. The salt was thoroughly dried by heating it to dull redness in a platinum dish. 74.59 grams were dissolved and diluted to a liter.

THE STANDARD SODIUM CHLORIDE.

The ordinary c. p. NaCl was treated as the KCl, and 58.5 grams were dissolved in a liter.

AMMONIUM NITRATE, NH_4NO_3.

This salt was obtained by neutralizing the c. p. HNO_3 of Baker & Adamson with their c. p. NH_4OH. It was recrystallized and found to be pure. The beautifully formed crystals were dried in the air-bath and then kept for some time over concentrated H_2SO_4. The weight became constant and 80.10 grams were made into a liter.

MAGNESIUM CHLORIDE, $MgCl_2$.

The ordinary c. p. $MgCl_2$ was recrystallized, and a solu-

[1] This is the value given by Kohlrausch for NaCl. The value of KCl referred to mercury through NaCl was found to be 918, which is also the value given by Kohlrausch n the Leitfaden, 1896.

tion made up with sp. gr. 1.1161 $\left(\frac{15°}{15°}\right)$, which, according to Gerlach's table, should contain 13.343 per cent. of salt in 100 parts by weight of the solution. A normal solution requires 47.6 grams of the compound per liter. This would require 356.74 grams of the foregoing solution. This amount was diluted to a liter and its sp. gr. $\left(\frac{18°}{4°}\right)$ was found to be 1.0373.

Two portions of 10 c. c. each were taken for analysis and determined as $Mg_2P_2O_7$ with the following results:

I. Wt. of $Mg_2P_2O_7$ found 0.5414
II. " " " " " 0.5409 } 0.5412 mean value.

This would correspond to 46.275 grams of $MgCl_2$ per liter instead of 47.6.

The solution was now concentrated until its sp. gr. by the Mohr balance became 1.0380, on the assumption that the value 1.0379 of Kohlrausch is correct.

A duplicate analysis gave the following values:

I. Wt. of $Mg_2P_2O_7$ 0.5580
II. " " " " 0.5557 } 0.5569 mean value.

Theoretrical value 0.5566.

This solution was taken to be normal.

Magnesium Sulphate, $MgSO_4$.

Trommsdorff's c. p. $MgSO_4$ was recrystallized and a solution made with sp. gr. 1.0573 according to the Mohr balance; 10 c. c. were taken for analysis and determined as $BaSO_4$.

Wt. of $BaSO_4$ found 1.1654 grm.
" " (theoretical) 1.1657 "

Its sp. gr. by the pyknometer was 1.0572.

Sodium Ammonium Hydrogen Phosphate, $Na(NH_4)HPO_4$.

The c. p. $Na(NH_4)HPO_4$ of Merck & Co. was recrystallized, dried with filter paper, and then in an air bath at a temperature a little above room temperature. This salt has $4H_2O$ of crystallization, and requires 69.71 grams per liter. It was made up by direct weight. Its sp. gr. was found to be 1.0345, which corresponds to the value found by Loomis.

DISODIUM HYDROGEN PHOSPHATE, Na_2HPO_4.

The ordinary c. p. Na_2HPO_4 was recrystallized, and after heating to about 120° C. until the weight became constant, a normal solution was prepared by dissolving 47.38 grams in a liter.

POTASSIUM DIHYDROGEN PHOSPHATE, KH_2PO_4.

The ordinary c. p. salt was recrystallized and dried at 100°. The normal solution was prepared by dissolving 45.39 grams in a liter.

PHOSPHORIC ACID, H_3PO_4.

The c. p. H_3PO_4 of Trommsdorff was tested and indicated no impurities. An approximate normal solution was prepared according to its sp. gr. Two independent analyses gave the following strength:

I. 10 c. c. of the solution were taken and the acid determined as $Mg_2P_2O_7$.

H_3PO_4 found 0.3276.

" (theoretical) 0.3268.

This indicated that the acid was 0.25 per cent. too strong.

II. The acid was titrated with standard alkali and $BaCl_2$ with rosolic acid as indicator.

This gave 100.20 per cent.

Average strength of acid 100.225.

The requisite amount of water was added to make the solution normal. Sp. gr. 1.0165.

OXALIC ACID, $(COOH)_2,2H_2O$.

The c. p. acid of Merck & Co. was employed for the preparation of the solution. 63.01 grams of the crystalline acid (= 45.01 grams of the anhydrous) are required per liter. Owing to its uncertain state of hydration, no attempt was made to dry it *thoroughly* but the crystalline compound was used and chemical analysis resorted to in order to determine its strength. Accordingly, 63 grams of the acid were weighed out, dissolved in a liter and titrated with potassium permanganate whose strength was determined just before using by balancing against a solution of ferrous sulphate made from iron wire. It was considered that the wire was 99.7 per cent. iron. This indicated that the solution required about

a gram of acid to bring it to normal strength. This amount was added and three independent analyses gave the following :

I. Titration with $K_2Mn_2O_8$, 100.08 %
II. Titration with NaOH, 100.15 % } 100.05 mean.
III. Determination as CaO, 99.93 %

No correction was applied.

Citric Acid, $C_3H_4OH(COOH)_3$.

This is a tribasic acid and crystallizes with one molecule of H_2O. The normal solution was prepared by balancing against normal NaOH with phenolphthalein as indicator.

Sodium Hydroxide, NaOH.

The normal solution of NaOH was prepared by taking metallic sodium, paring off its crust in a cold room, with open windows, then placing it in a large silver dish, setting dish on a tripod underneath a bell-jar, together with a vessel of water containing a half-submerged sponge. A rubber stopper through which glass tubes passed was thrust into the mouth of the jar, and these tubes connected with U-shaped tubes containing soda-lime. This permits a free circulation of air, allows for the escape of hydrogen, and also absorbs the CO_2 contained in the air. Without this precaution there is danger of an explosion. By standing the apparatus in a warm room, all of the sodium is quietly changed into NaOH in the course of a few days. The process is slow, but one is rewarded with a pure product of NaOH. The normal solution was prepared as follows :

In a well aired room, about 50 c. c. of the NaOH were transferred to a 200 c. c. flask which contained about 150 c. c. of H_2O. This solution contained more NaOH than was required for a liter. The greater part of it was poured into a beaker containing about 700 c. c. of H_2O, and the sp. gr. determined by the Mohr balance. It was then diluted to the approximate strength. It was finally brought to the proper strength by balancing against standard acid. The normal solution was thus prepared in a short time, and before the NaOH could materially affect the glass or take up any appreciable quantity of CO_2. According to my experience the sp. gr. of a normal so-

Table II.

Values of K multiplied by 10^{10} *at* 18°.

m	NH_4NO_3	$\frac{1}{2}MgCl_2$	$\frac{1}{2}MgSO_4$	$\frac{1}{3}NaNH_4HPO_4$	$\frac{1}{3}Na_2HPO_4$	$\frac{1}{3}KH_2PO_4$	$\frac{1}{3}H_3PO_4$	$\frac{1}{2}(COOH)_2$	$\frac{1}{3}C_3H_4OH(COOH)_3$	NaOH	v
	$K_{18}\cdot10^{10}$.	$K_{18}\cdot10^{10}$.	$K_{18}\cdot10^{10}$.	$K_{18}\cdot10^{10}$.	$K_{18}\cdot10^{10}$.	$K_{18}\cdot10^{10}$.	$K_{18}\cdot10^{10}$.	$K_{18}\cdot10^{10}$.	$K_{18}\cdot10^{10}$.	$K_{18}\cdot10^{10}$.	
(Water)	1.38	1.80	1.05	1.64	1.21	1.13	1.89	1.59	1.45	1.58	
0.0001017	13.38	12.75	11.64	6.31	6.17	4.30	11.47	23.83	14.46	———	10000.
0.0002032	25.35	23.39	21.89	12.65	11.85	7.42	21.09	44.35	24.66	———	5000.
0.0006077	72.63	64.75	59.61	36.40	34.27	19.78	61.02	112.10	57.53	103.0	1666.6
0.001009	119.02	105.27	95.34	60.41	56.32	31.08	100.94	170.94	84.92	185.3	1000.
0.002015	233.29	205.20	177.99	118.89	110.00	59.96	195.80	327.80	140.94	387.1	500.
0.006024	675.30	585.80	464.35	339.51	314.16	170.88	525.41	918.05	290.69	1152.3	166.6
0.01000	1101.33	948.00	713.00	548.20	506.40	277.00	801.70	1483.60	400.15	1906.0	100.
0.03	3174.0	2628.	1750.	1522.	1410.	792.	1880.	4016.	779.	5646.	33.3
0.05	5748.5	4202.	2650.	2407.	2243.	1300.	2735.	6214.	1030.	9310.	20.
0.10	9973.8	7939.	4630.	4441.	4111.	2501.	4380.	10940.	1504.	18280.	10.
0.50	44215.0	33455.	16440.	16974.	15632.	10838.	12745.	35500.	3434.	81440.	2.
1.00	83040.0	59346.	27020.	28280.	26183.	19990.	20770.	55570.	5020.	146900.	1.

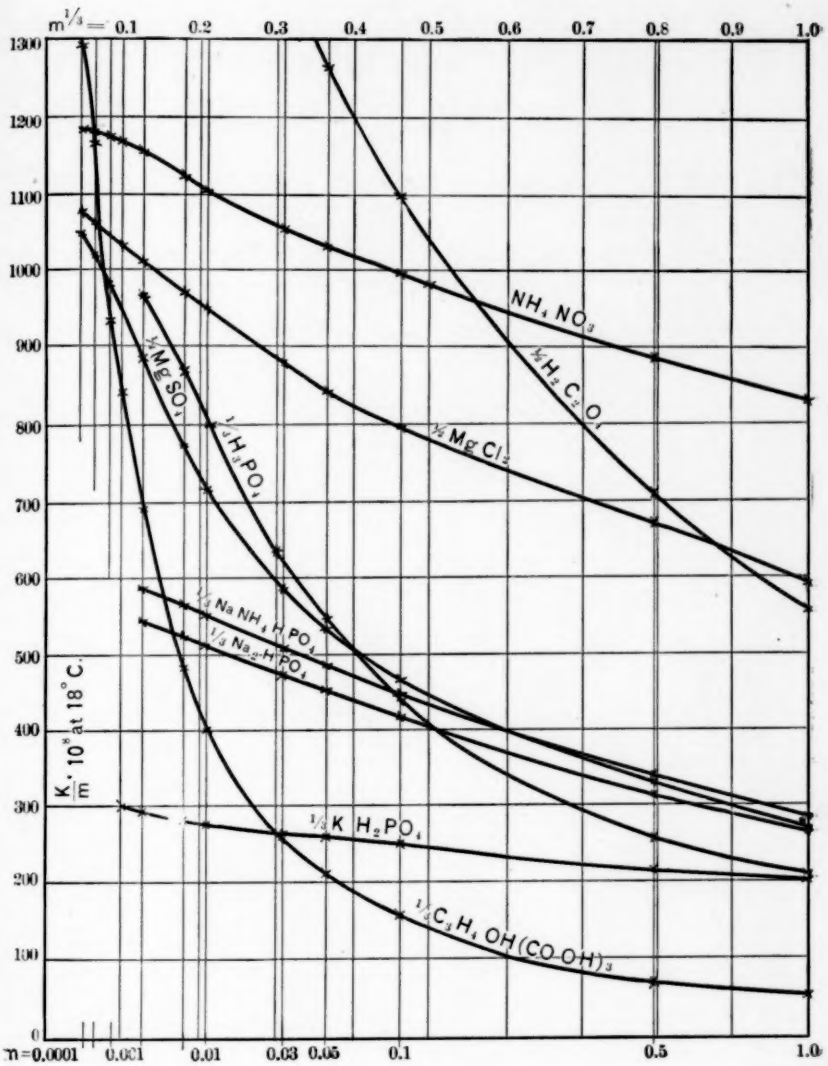

m^1/3 = 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
K, / m 10^5 at 18° C.
n=0.0001 0.001 0.01 0.03 0.05 0.1 0.5 1.0
NH4 NO3
½ H2 C2 O4
½ Mg Cl2
½ Mg SO4
1/4 H3 PO4
1/3 Na NH4 H PO4
1/3 Na2 H PO4
½ K H2 PO4
1/4 C3 H4 OH(CO OH)3

Fig. 3.

Table III.

m	NH_4NO_3		$\frac{1}{2}MgCl_2$		$\frac{1}{2}MgSO_4$		$\frac{1}{3}NaNH_4HPO_4$		$\frac{1}{3}Na_2HPO_4$		$\frac{1}{3}KH_2PO_4$		$\frac{1}{3}H_3PO_4$		$\frac{1}{2}(COOH)_2$		$\frac{1}{3}C_3H_4OH(COOH)_3$		NaOH		v
	$\frac{K}{m}\cdot 10^5$	$0.1069\times\frac{K}{m}\cdot 10^5$	$\frac{K}{m}\cdot 10^5$	$0.1069\times\frac{K}{m}\cdot 10^5$	$\frac{K}{m}\cdot 10^5$	$0.1069\times\frac{K}{m}\cdot 10^5$	$\frac{K}{m}\cdot 10^5$	$0.1069\times\frac{K}{m}\cdot 10^5$	$\frac{K}{m}\cdot 10^5$	$0.1069\times\frac{K}{m}\cdot 10^5$	$\frac{K}{m}\cdot 10^5$	$0.1069\times\frac{K}{m}\cdot 10^5$	$\frac{K}{m}\cdot 10^5$	$0.1069\times\frac{K}{m}\cdot 10^5$	$\frac{K}{m}\cdot 10^5$	$0.1069\times\frac{K}{m}\cdot 10^5$	$\frac{K}{m}\cdot 10^5$	$0.1069\times\frac{K}{m}\cdot 10^5$	$\frac{K}{m}\cdot 10^5$	$0.1069\times\frac{K}{m}\cdot 10^5$	
0.0001	1180	126.1	1077	115.1	1041	111.4	(459)	——	(488)	——	(312)	——	(942)		2200	235.2	1280	136.8	——		10000
0.0002	1179	126.0	1062	113.5	1021	109.1	(542)	——	(523)	——	(309)	——	(945)		2100	224.5	1142	122.1	——		5000
0.0006	1172	125.3	1036	110.7	962	102.8	(572)	——	(544)	——	(307)	——	(973)		1820	194.5	923	98.7	——		1666.6
0.001	1165	124.5	1026	109.7	935	100.0	(582)	——	(546)	58.4	297	31.7	982	105.0	1690	180.7	827	88.4	(1820)		1000
0.002	1151	123.0	1009	107.9	878	93.8	582	62.2	540	57.7	292	31.2	962	102.8	1610	172.1	692	74.0	1913	204.5	500
0.006	1119	119.6	969	103.6	769	82.2	561	60.0	519	55.5	282	30.1	869	92.9	1520	162.5	482	51.5	1910	204.2	166.6
0.01	1100	117.6	948	101.3	713	76.1	546	58.4	505	54.0	276	29.5	799	85.4	1480	158.2	399	42.7	1904	203.5	100
0.01 (direct)	1108	118.4	949	101.4	713	76.1	545	58.3	504	53.9	275	29.4	790	84.5	1480	158.2	396	42.3	1902	203.3	100
0.03	1057	113.0	876	93.6	583	63.3	507	54.2	470	50.2	264	28.2	627	67.0	1339	143.1	260	27.8	1882	201.2	33.3
0.05	1029	110.0	840	89.8	530	56.6	481	51.4	449	48.0	260	27.8	547	58.5	1243	132.9	206	22.0	1862	199.0	20
0.10	997	106.6	794	84.7	463	49.5	444	47.5	411	44.0	250	26.7	438	46.8	1094	116.9	150.4	16.1	1828	195.4	10
0.50	884	94.5	669	71.5	329	35.2	339.5	36.3	313	33.5	217	23.2	255	27.3	710	75.9	68.7	7.3	1629	174.1	2
1.00	830.4	88.8	593.5	63.4	270	28.9	283	30.3	262	28.0	200	21.4	207.7	22.2	555.7	59.4	50.2	5.4	1469	157.0	1

lution of NaOH changes much more rapidly than the electrical conductivity. Thus a solution made up on January 25th had a sp. gr. of 1.0419; but three days later it had increased to 1.0421. On January 25th the same solution gave $K.10^7 = 146.6$, while four days later it was 146.8.[1]

THE EXPERIMENTAL RESULTS.

Table II. gives the conductivity of the electrolytes in aqueous solution referred to the mercury unit.

Table III. shows the equivalent-conductivity $\frac{K}{m}$ multiplied by 10^8. For convenience these values are reduced to the new basis by multiplying by the factor 0.1069.

GRAPHIC REPRESENTATION OF THE CONDUCTIVITY.

In Fig. 3, $\frac{K}{m} \cdot 10^8$ is represented by ordinates and the cube root of the concentration ($m^{\frac{1}{3}}$) is represented by abscissas. The curve for NaOH is not given, and only a part of the oxalic acid curve is represented. By referring to Table II. it will be seen that the specific conductivity K increases as the concentration of the solution increases, but they do not increase at the same rate. K lags more and more behind as m increases. Table III. shows that the equivalent-conductivity becomes less as the concentration becomes greater. $\frac{K}{m}$ reaches a maximum value, in general, in the most dilute solution. This is well shown by the curves.

THE TEMPERATURE-COEFFICIENT C.

$C = \frac{1}{K} \frac{K_1 - K_2}{t_1 - t_2}$ is the general formula for the temperature-coefficient. To express the mean temperature coefficient between t_1 and t_2, in parts of the conductivity at 18° when $t_1 = 26°$ and $t_2 = 18°$, we make use of the following formula:

$$C_{22} = \frac{1}{K_{18}} \left(\frac{du}{dt} \right)_{22}, \text{ or } C_{22} = \frac{1}{K_{18}} \left(\frac{K_{26} - K_{18}}{8} \right).$$

The values of C given in Table IV. are for the normal solutions.

[1] McLauchlan, Princeton University Bulletin, January, 1899.

TABLE IV.

Value of C.		Value of C.		Value of C.	
NH_4NO_3	0.0201	Na_2HPO_4	0.0257	$(COOH)_2$	0.0141
$MgCl_2$	0.0221	KH_2PO_4	0.0220	$C_3H_4OH(COOH)_3$	0.0109_5
$MgSO_4$	0.0224	H_3PO_4	0.0103	NaOH	0.0208
$NaNH_4HPO_4$	0.0236				

The values of C for the salts lie near together, while those of the acids are much smaller.

THE "DEGREE OF DISSOCIATION" $\frac{\mu_x}{\mu_\infty}$

The "degree of dissociation," or ionization, is found by dividing the equivalent conductivity of any solution (μ_x) by its equivalent conductivity in infinite dilution (μ_∞). These values are given in Table V.

TABLE V.

Values of $\frac{\mu_x}{\mu_\infty}$.

m	NH_4NO_3	$\frac{1}{2}MgCl_2$	$\frac{1}{2}MgSO_4$	$\frac{1}{2}NaNH_4HPO_4$	$\frac{1}{2}NaHPO_4$	$\frac{1}{3}KH_2PO_4$	$\frac{1}{3}H_3PO_4$	$\frac{1}{2}(COOH)_2$	$\frac{1}{3}C_3H_4OH(COOH)_3$	NaOH	v
0.0001	(1.000)	0.988	0.946	——	——	——	——	0.621	0.366	——	10000
0.0002	(0.999)	.974	.928	——	——	——	——	.593	.324	——	5000
0.0006	.993	.950	.874	——	——	——	——	.511	.264	——	1666.6
0.001	.987	.941	.850	——	——	0.990	0.869	.477	.237	——	1000.
0.002	.975	.926	.798	0.954	0.964	.973	.851	.455	.198	0.929	500.
0.006	.948	.889	.699	.920	.927	.940	.769	.429	.138	.927	166.6
0.01	.932	.870	.648	.895	.902	.920	.707	.418	.114	.923	100.
0.03	.896	.804	.530	.831	.839	.880	.555	.378	.0745	.907	33.3
0.05	.872	.771	.482	.789	.802	.867	.484	.351	.0599	.898	20
0.10	.845	.728	.420	.728	.734	.833	.388	.309	.0431	.881	10
0.50	.749	.614	.299	.556	.559	.723	.226	.201	.0197	.785	2
1.00	.704	.545	.245	.464	.468	.667	.184	.157	.0144	.713	1

The following limiting values (μ) were used in calculating the "degree of dissociation":

NH_4NO_3, 1180; $MgCl_2$, 1090; $MgSO_4$, 1100; $NaNH_4HPO_4$, 610; Na_2HPO_4, 560; KH_2PO_4, 300; H_3PO_4, 1130; $(COOH)_2$, 3540; $C_3H_4OH(COOH)_3$, 3490; NaOH, 2060. The evidence in favor of these values will be given later in the discussion of the compounds.

A comparison of $\frac{\mu_x}{\mu_\infty}$ with the degree of dissociation δ as deduced from the depression of the freezing points (Δ) as found by Loomis is given in Table VI. δ was calculated from the formula

$$\frac{\Delta}{m} = 1.86\ (1 + k\delta),^1$$

in which m as used by Loomis is the concentration in *gram-molecules* per liter and k denotes the number of parts into which the electrolyte dissociates.

1.86 is used for the value of the van't Hoff constant throughout this work. Loomis has much experimental evidence in favor of this number which, however, is not ready for publication, but which has been kindly furnished me for the purpose of this paper.

DISCUSSION OF RESULTS.

I. THE NEUTRAL SALTS.

1. *Ammonium Nitrate, NH_4NO_3.*

Ammonium nitrate is the best conductor of the neutral salts measured. It appears to be quite completely dissociated[2] into its ions NH_4 and NO_3 when $m = 0.0001$. The limiting value μ_∞ was regarded as 1180, this being the highest value found. A second measurement gave the same values for $\frac{K}{m} \cdot 10^8$. The sum of the independent migration velocities of the ions ($\mu_\infty = u + v$) for this salt, adopting 600 for NH_4 and 580 for NO_3[3], is 1180.

By referring to Table VI. it will be seen that the values of $\frac{\mu_x}{\mu}$

[1] Zeit. Phys. Chem., Vol. I. (Arrhenius).

[2] The following discussion is made upon the hypothesis of the Dissociation Theory, it being regarded the most probable theory thus far advanced and thus affording the best working basis.

[3] Leitfaden, 1896, p. 483.

TABLE VI.

m	NH_4NO_3		$\frac{1}{2}MgCl_2$		$\frac{1}{2}MgSO_4$		$\frac{1}{3}NaNH_4HPO_4$		$\frac{1}{3}Na_2HPO_4$		$\frac{1}{3}KH_2PO_4$		$\frac{1}{3}H_3PO_4$		$\frac{1}{2}(COOH)_2$		$\frac{1}{3}C_3H_4OH(COOH)_3$		NaOH	
	$\frac{\mu_x}{\mu_0}$ (F.)	δ (L.)	$\frac{\mu_x}{\mu_0}$ (F.)	δ (L.)	$\frac{\mu_x}{\mu_0}$ (F.)	δ (L.)	$\frac{\mu_x}{\mu_0}$ (F.)	δ (L.)	$\frac{\mu_x}{\mu_0}$ (F.)	δ (L.)	$\frac{\mu_x}{\mu_0}$ (F.)	δ (L.)	$\frac{\mu_x}{\mu_0}$ (F.)	δ (L.)	$\frac{\mu_x}{\mu_0}$ (F.)	δ (L.)	$\frac{\mu_x}{\mu_0}$ (F.)	δ (L.)	$\frac{\mu_x}{\mu_0}$ (F.)	δ (L.)
0.01	0.932	0.93																		
0.02	.914	.90	0.837	0.88	0.574	0.51									0.398	0.38			0.916	0.86
0.03							0.831	0.83	0.839	0.84	0.880	0.93	0.555	0.52			0.075	0.07		
0.04			.787	.86	.500	.40									.365	.36				
0.05	.872	.87																	.898	.86
0.06							.777	.79	.788	.80	.860	.94	.460	.44			.054	.05		
0.10	.845	.84	.728	.84	.420	.30									.309	.32			.881	.83
0.15							.707	.71	.712	.74	.820	.87	.345	.33			.036	.04		
0.20	.805	.79	.700	.83	.373	.20									.282	.27			.857	.83
0.30			.651	.84			.642	.64	.646	.67	.780	.81	.274	.27			.026	.03		
0.40			.642	.85	.320	.11									.228	.22				
0.50			.614	.87																
0.60			.585	.89			.538	.55			.713	.73	.217	.21			.019	.02		

agree well with the value of δ as deduced from the depression of the freezing points as found by Loomis. For $m = 0.01$ these values are 0.932 and 0.93, respectively. The difference is less than the probable error of the Loomis method of determining the freezing points of solutions. The agreement for the concentrated solutions is also marked. The average difference is 0.7 per cent.

Kohlrausch[1] has measured the conductivity of NH_4NO_3 from $m = 0.5$ to $m = 8$. For $m = 0.5$ he finds $\frac{K}{m} \cdot 10^8 = 884$, and for $m = 1$, 831. These values correspond with those given in Table III., the curves meeting in this region.

2. *Magnesium Chloride, $MgCl_2$.*

It is assumed that $MgCl_2$ dissociates into three parts. The highest value for μ was found to be 1077, but the curve, when $m = 0.0001$, is still rising, so 1090 ($\frac{1}{2}$Mg = 460 and Cl = 630) was regarded as the most probable value of μ_∞. According to the depression of the freezing-point as found by Loomis in the solution $m = 0.02$, $\delta = 0.88$. The corresponding value calculated from the conductivity is 0.837. These agree fairly well at this point and would no doubt agree more closely if the measurements of Δ could be made accurately in still more dilute solutions. For the more concentrated solutions, however, the value of $\frac{\mu_x}{\mu_\infty}$ and δ differ by a greater amount. Loomis[2] found that the *molecular depression* $\frac{\Delta}{m}$ is 5.14 for $= m$ 0.01. This value decreased for a time, reached a *minimum*, and finally *increased* with concentration. No explanation has been given, so we would not expect the values as found by the conductivity to agree with those found by Loomis, especially as there is nothing abnormal in the conductivity of $MgCl_2$ as the concentration increases. Ostwald,[3] Kohlrausch[4] and Walden[5] have also made measurements for some of the concentrations of $MgCl_2$. Ost-

[1] Wied. Ann., Vol. VI., p. 149.
[2] PHYS. REVIEW, Vol. III., No. 16, pages 277 and 281.
[3] Phys. Chemie, Vol I., p. 109.
[4] Weid. Ann., Vol. XXVI., p. 194.
[5] Phys. Chemie, I., 529, 1887.

wald and Walden made their measurements at 25°C. and without pushing the measurements to extreme dilution, while Kohlrausch conducted his measurements at 18° and from $m = 0.01$ to $m = 10$. These values agree well with the corresponding values in Table III.

3. *Magnesium Sulphate, $MgSO_4$.*

Kohlrausch,[1] Ostwald[2] and others have measured the conductivity of $MgSO_4$, so the present measurements were made more especially for testing the resistance vessel. Loomis also used the same normal solution for a new determination of Λ.

The values for $\frac{K}{m}$ are compared with those found by Kohlrausch.[3]

m	$\frac{K}{m} \cdot 10^8$ (Foster.)	(Kohlrausch.)
0.0001	1041	1034
.0002	1021	1015
.0006	962	967
.001	935	935
.002	873	881
.006	769	773
.01	713	715
.03	583	587
.05	530	532
.10	463	467
.50	329	330
1.00	270	271

The limiting value of $\frac{K}{m}$ was regarded as 1100. Kohlrausch gives its probable value as 1090, and $u + v$ from the probable velocity of the ions is 1120 ($\frac{1}{2}Mg = 460$ and $\frac{1}{2}SO_4 = 660$).

By an extension of the curve of $MgSO_4$ it is quite certain that μ_∞ is not far from 1100, which is a little greater than the corresponding number for $MgCl_2$. This electrolyte is less than one-half as much dissociated in the normal solution as $MgCl_2$. Professor Loomis has recently repeated his work on the freezing-points of

[1] Wied. Ann., Vol. XXVI., p. 196.
[2] Zeit. Phys. Chemie, Vol. I., p. 109.
[3] Leitfaden, 1896, p. 483.

solutions of $MgSO_4$, and the values for δ, taking his new experimental value of the van't Hoff constant as 1.86, are given in Table VI. For $m = 0.02$ the value of δ is 0.51, while the conductivity makes it 0.574. The values, however, for $\frac{\mu_v}{\mu_\infty}$ and δ are seen in Table VI. to be rapidly converging as the dilution increases, and there is no evidence in the experimental data thus far obtained to show that the values would not be identical in *extreme* dilution.

The neutral salts here studied give two groups of electrolytes:

I. NH_4NO_3—an electrolyte containing univalent basic and acid radicals.

II. $MgCl_2$ and $MgSO_4$—electrolytes containing bivalent radicals.

It is to be observed:

1. The NH_4NO_3 is highly dissociated and μ_∞ is practically reached in the dilute solutions. It therefore complies very nearly with the law of Kohlrausch.

2. The rate at which $\frac{\mu_v}{\mu_\infty}$ increases with dilution is greater for $MgCl_2$ and $MgSO_4$ than for NH_4NO_3. This is very marked in the case of $MgSO_4$ after the dilution is carried to $m = 0.05$, where the curve rises abruptly after the manner of H_3PO_4 or $C_3H_4OH(COOH)_3$.

II. THE PHOSPHATES.

1. Sodium Ammonium Hydrogen Phosphate, $NaNH_4HPO_4$.

The curve of this salt rises gradually and runs approximately parallel to the NH_4NO_3 curve, but much below it. When the dilution is carried into the region $m = 0.001$, the curve no longer rises, but suddenly begins to drop, after the manner of the curves of acids and bases. By extending the curve, however, the approximate values for μ_∞ can be obtained. This was regarded as 610 for $NaNH_4HPO_4$. This value is confirmed by the freezing points as determined by Loomis. Thus for $m = 0.03$ the value of $\frac{\mu_v}{\mu_\infty}$ is 0.831 and δ is 0.83. The values for the more concentrated solutions agree well with one another, the average difference amounting to 0.9%. It is assumed that this salt dissociates into three parts, Na—NH_4—HPO_4.

2. Disodium Hydrogen Phosphate, Na_2HPO_4.

This salt is analogous to the one just considered in every respect, and its behavior might have been predicted from that of $NaNH_4HPO_4$.

The curves run parallel, but the conductivity of $NaNH_4HPO_4$ is a little greater. This would be anticipated from the independent migration velocities of the ions—the NH_4 radical having a greater velocity than the Na ion. The Na_2HPO_4 curve likewise drops in dilute solutions.

By extending it, the value of μ_∞ was taken to be 560.

For $m = 0.03$, the value of $\frac{\mu_x}{\mu_\infty}$ is thus 0.839, while Loomis found for δ the value 0.84. The average agreement is within 1.6%. It is assumed that this electrolyte dissociates into three parts, Na—Na and HPO_4. Measurements have been made also by M. Berthelot,[1] Walden[2] and M. Bouty.[3]

3. Potassium Dihydrogen Phosphate, KH_2PO_4.

The curve of KH_2PO_4 rises very gradually and runs very much lower than the curves of the phosphates just considered. This curve does not drop in dilute solutions; but, on the other hand, it *rises* to a certain extent, the values of $\frac{K}{m}$ becoming somewhat larger after the dilute solutions have stood a few minutes. As the specific conductivity of the very dilute solutions is not very large compared to that of the water, this increase in conductivity may be partly due to the matter dissolved by the water. It is said, too, that the phosphates are not very stable in dilute solutions.

By extending the curve, 300 was regarded as the most probable value of μ_∞. The values of $\frac{\mu_x}{\mu_\infty}$ are found to be much lower than the values of δ as found by Loomis. These values are given in Table VI. For $m = 0.03$, they are 0.88 and 0.93, respectively. The average difference is about 4.5%. It is assumed that this salt

[1] Annales de Chem. et de Phys., T. XXVIII., 1893.
[2] Ph. Chem., 8, 433, 1891.
[3] Annales de Chem. et de Phys., T. XIV., 1884.

dissociates into two parts, $K-H_2PO_4$. Some of the measurements of KH_2PO_4 have been previously made by Berthelot[1] and Bouty.[2]

4. *Phosphoric Acid, H_3PO_4.*

The curve of phosphoric acid starts in the region of that of KH_2PO_4; it rises gradually until m is about 0.1, then very rapidly until m is about 0.001, after which it makes a rather sudden drop. The value of μ_∞, by an extension of the curve, was taken to be 1130. Kohlrausch gives it as 1100, and Berthelot makes it about 1125 at 17°C.

In Table VI. it may be seen that the values of $\frac{\mu_v}{\mu_\infty}$, the degree of dissociation, closely confirm the values found from the depression of the freezing-point. For $m = 0.03$ the difference amounts to 3.5%, while the average difference is 1.6%.

It is assumed that H_3PO_4 dissociates into two parts, giving $H - H_2PO_4$. Its conductivity has been determined by Ostwald,[3] Kohlrausch[4] and Berthelot.[5]

A study of the phosphates shows:

1. That the degree of dissociation of $NaNH_4HPO_4$ and Na_2HPO_4 is the same (see values in Table VI.), while that of KH_2PO_4 is greater and that of H_3PO_4 is less.

2. The degree of dissociation of the acid phosphates is comparable to that of neutral salts, while the equivalent-conductivity is much inferior.

3. As the H of the H_3PO_4 is replaced by a univalent metallic radical, the conductivity is increased. Loomis found that the value of Δ also increased.

4. The degree of dissociation of KH_2PO_4 is comparable to that of NH_4NO_3, KCl, NaCl and other compounds containing univalent radicles and the depressions of the freezing-point of these solutions point out the same fact. This appears to be proof that all of these electrolytes dissociate into two parts.

[1] Ann. de Chem. et de Phys., T. XXVIII., 1893.
[2] Ann. de Chem. et de Phys., T. XIV., 1884.
[3] Journ. für Chem., Vol. XXXI., p. 460.
[4] Leitvermögen der Elektrolyte, p. 160.
[5] Ann. de Chem. et de Phys., T. XXVIII.

5. The $NaNH_4HPO_4$ and Na_2HPO_4 appear to be dissociated into three parts each, for the calculated values of the freezing-points upon this assumption agree closely with the values actually observed by Loomis.

6. The equivalent conductivity of H_3PO_4 is much less than that of other acids, and reaches about the same maximum value as the neutral salts.

III. The Organic Acids.

1. *Oxalic Acid* $(COOH)_2$.

The curve for oxalic acid begins in the region of that of $MgCl_2$, but when $m = 0.15$ it passes above the curves of the neutral salts and rises very rapidly. Owing to the unstability of the acid in dilute solution, it is impossible to experimentally determine the value of μ_∞. This was calculated, however, from the following equation: μ_∞ for acid $= \mu_\infty HCl - \mu_\infty KCl + \mu_\infty KR$,[1] in which $\mu_\infty KR$ stands for the limiting value of the potassium salt of the acid under consideration. The conductivity of $(COOK)_2$[2] has been determined by MacGregory. The highest value found was 1172. Taking μ_∞ HCl as 3600 and μ_∞ KCl as 1230, we have 3540 as the value for $\mu_\infty (COOH)_2$ at 18°.

Regarding the velocity of $\frac{1}{2}C_2O_4$ as 590[3] and that of H as 2975, the value of $\mu_\infty (COOH)_2$ would be 3565. 3540 was used for calculating $\frac{\mu_v}{\mu_\infty}$.

The depressions of the freezing-point as found by Loomis point to the same limiting value of μ, as will be seen by reference to Table VI.

Thus for $m = 0.02$, the difference is 1.8%. The average difference is 1.1%.

Measurements of oxalic acid have been made by Ostwald[4] and Berthelot.[5]

[1] Zeit. für Phys. Chem., 16, 1895, p. 83.
[2] Wied. Ann., 51, 126, 1894.
[3] Kohhausch & Holborn, Leitvermögen der Elektrolyte, p. 200.
[4] Jour. für Chemie, Vol. XXXI., p. 457.
[5] Annal. de Chemie, Vol. XXI., p. 49.

2. *Citric Acid*, $C_3H_4OH(COOH)_3$.

This is a tribasic acid and may be represented by the following formula:

$$\begin{array}{l} CH_2-COOH \\ | \\ COH-COOH \\ | \\ CH_2-COOH. \end{array}$$

This acid is a very poor conductor in concentrated solutions. When m reaches about 0.05 the values of the equivalent-conductivity begin to rise and soon pass above those of the neutral salts.

In dilute solutions, citric acid is relatively a good conductor. The value for μ_∞ was calculated in a manner similar to that for $(COOH)_2$. Thus, the value of μ_∞ for potassium citrate according to Berthelot is 1120.[1] Accordingly μ_∞ citric acid = 3490 at 18°. This value is confirmed by the value of the freezing-points as shown in Table VI. For $m = 0.03$, the difference amounts to only 0.5%. The average difference between $\frac{\mu_v}{\mu_\infty}$ and δ is about 0.4%.

The degree of dissociation of citric acid is very small in concentrated solutions. For $m = 1$ it conducts only about $\frac{1}{10}$ as well as oxalic acid, and for $m = 0.0001$, about ½ as well.

In extreme dilution, these curves lie near each other. Berthelot has measured the conductivity of some of the concentrations of citric acid.

IV. SODIUM HYDROXIDE *NaOH*.

This electrolyte is one of the best conductors. In concentrated solutions, it conducts much better than any other electrolyte discussed in this paper. Its conductivity changes slowly with dilution, soon reaching a maximum value, and then begins to decrease. The values of $\frac{K}{m}$ are certainly not trustworthy beyond $m = 0.002$. It being impossible, therefore, to determine μ_∞ experimentally, the sum of the ionic velocities (Na = 410 and OH = 1650) was taken to represent it. This gives 2060. By referring again to Table VI., it will be seen that the values of $\frac{\mu_v}{\mu_\infty}$ do not agree with δ as de-

[1] Thèses a la Faculte des Science de Paris, 1891.

duced from the depressions of the freezing-point; and, what is especially peculiar, the disagreement becomes less as the concentration of the solution *increases*. The same remark is known to apply to KOH, HCl and HNO_3.[1]

No explanation has yet been given for this disagreement. It is too large to be explained by any known source of experimental error in the method by which Loomis determines the freezing-points of solutions.

Measurements of the conductivity of NaOH have been made by Kohlrausch[2] and Ostwald.[3] All of these values agree closely in dilute solutions and fairly well in the most concentrated solutions. In solutions of medium dilution the disagreement is greater. The present values are about as much larger than those found by Ostwald as Ostwald's are larger than those found by Kohlrausch.

Concluding Remarks.

In summing up the work thus far it may be seen that the values of the depressions of the freezing-point as determined by Loomis are confirmed, in general, within as narrow limits as one should expect; and in many instances the agreement is so close as to be pronounced "accidental." The results also, with only a few minor exceptions, conform to the dissociation theory set forth by Arrhenius. In the present stage of physical chemistry, we are compelled to look upon this theory as the one which most nearly corresponds with the experimental data, thus affording at least a working hypothesis by means of which the most important generalizations can be made.

In conclusion, I wish to express my gratitude to Professor Loomis who has given me much valuable assistance throughout this investigation.

Phys. Lab., Princeton University, March, 1899.

[1] Loomis, Phys. Rev., Vol. III., No. 16, 1896; and Vol. IV., 1897.
[2] Wied. Ann., 26, 1885, and Leitvermögen der Elek., 1898.
[3] Journ. für Chemie, Vol. XXXIII., p. 355.

ON THE DERIVATION OF THE EQUATIONS OF A PLANE ELECTRO-MAGNETIC WAVE.

BY EDWARD B. ROSA.

THE derivations of the equations of an electro-magnetic wave from the general equations of the field is straightforward, and the information deduced therefrom is important. That the magnetic and electric forces are at right angles to each other, and both in the wave front; that the velocity of propagation is inversely as the square root of the product of the dielectric constant and the magnetic permeability; that the energy is half electro-static and half electro-magnetic; that the magnetic and electric waves are in phase with each other; these and other facts are neatly and indisputably proved by the equations. But when all that has been done, the average student is apt to wonder how so much information can be extracted from a lot of differential equations. The physical relations are not as clearly perceived as are the mathematical, and there remains a feeling of dissatisfaction with the demonstration. This can be cleared away very largely by constructing the wave and demonstrating its properties from fundamental principles, disregarding the general equations. That is to say, instead of establishing the general equations and deducing therefrom the case of a plane wave, let the plane wave be derived from elementary fundamental principles. This will give the same results and illuminate the process of the wave. I would not recommend this proceeding as a substitute for the other, but as confirming and illustrating the general demonstration it seems to me to be interesting and valuable.

It will be convenient first to put down the general equations of the field, and then very briefly the derivation of the equations of the wave. We shall then derive the wave independently, and compare the results step by step.

a, b, c are the components of the magnetic induction, B.

α, β, γ are the components of the magnetic force, H.

X, Y, Z are the components of the electric force, E.
u, v, w are the components of the electric current density, i.
f, g, h are the components of the electric displacement, D.
Then we have,

$$\left.\begin{aligned} 4\pi u &= \frac{d\gamma}{dy} - \frac{d\beta}{dz} \\ 4\pi v &= \frac{d\alpha}{dz} - \frac{d\gamma}{dx} \\ 4\pi w &= \frac{d\beta}{dx} - \frac{d\alpha}{dy} \end{aligned}\right\} \quad (1)$$

$$\left.\begin{aligned} -\mu\frac{d\alpha}{dt} &= \frac{dZ}{dy} - \frac{dY}{dz} \\ -\mu\frac{d\beta}{dt} &= \frac{dX}{dz} - \frac{dZ}{dx} \\ -\mu\frac{d\gamma}{dt} &= \frac{dY}{dx} - \frac{dX}{dy} \end{aligned}\right\} \quad (2)$$

If the specific resistance of the dielectric is infinite, the whole current is the displacement current, the time rate of change of the electric displacement or polarization. Thus $u = \frac{K}{4\pi}\cdot\frac{dX}{dt}$ etc., and equations (1) become

$$\left.\begin{aligned} K\frac{dX}{dt} &= \frac{d\gamma}{dy} - \frac{d\beta}{dz} \\ K\frac{dY}{dt} &= \frac{d\alpha}{dz} - \frac{d\gamma}{dx} \\ K\frac{dZ}{dt} &= \frac{d\beta}{dx} - \frac{d\alpha}{dy} \end{aligned}\right\}. \quad (3)$$

Differentiating the first equation of (3) with respect to t, and substituting in the right hand side from the equations (2) we get, after some reductions (see Electricity and Magnetism by J. J. Thomson),

$$\mu K\frac{d^2X}{dt^2} = \frac{d^2X}{dx^2} + \frac{d^2X}{dy^2} + \frac{d^2X}{dz^2},$$

and similarly two equations in Y and Z, which are the equations of the wave.

When the quantities are independent of the coördinates y and z we have, taking the Y compound of the force,

$$\mu K \frac{d^2Y}{dt^2} = \frac{d^2Y}{dx^2}. \tag{4}$$

The solution of this equation gives

$$Y = f_1\left(x - \frac{t}{\sqrt{\mu K}}\right) + f_1\left(x + \frac{t}{\sqrt{\mu K}}\right). \tag{5}$$

Similarly we can find the equation for the propagation of the magnetic force γ.

I. *Equation* (5) *expresses the fact that the force of* Y *is propagated in both directions along the axis of* X *with a velocity* $V = \frac{1}{\sqrt{\mu K}}$, the wave being a plane wave.

Assuming the electric force to vary as the sine of an angle we may write, since the displacement is proportional to the force,

$$\left.\begin{aligned} f &= f_0 \sin \frac{2\pi}{\lambda}(lx + my + nz - Vt) \\ g &= g_0 \sin \frac{2\pi}{\lambda}(lx + my + nz - Vt) \\ h &= h_0 \sin \frac{2\pi}{\lambda}(lx + my + nz - Vt) \end{aligned}\right\} \tag{6}$$

where f, g, h are the components of the displacement D, and V is the velocity, $\frac{1}{\sqrt{\mu K}}$. Since f, g and h obey the law of continuity,

$$\frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz} = 0.$$

It follows at once that $lf + mg + nh = 0$, where l, m, n are the direction cosines of the normal to the wave front. Hence,

II. *The electric displacement is perpendicular to the normal to the wave front; that is, it is in the wave front.*

Substituting in equation (2) the values of X, Y, Z in terms of f, g, h and integrating, it is readily proved that

$$\left.\begin{aligned} \alpha &= 4\pi V(ng - mh) \\ \beta &= 4\pi V(lh - nf) \\ \gamma &= 4\pi V(mf - lg) \end{aligned}\right\}. \tag{7}$$

Whence it follows at once that

$$l\alpha + m\beta + n\gamma = 0$$

and

$$f\alpha + g\beta + h\gamma = 0.$$

Therefore,

III. *The magnetic force is perpendicular to the normal to the wave front; and the electric and magnetic forces are perpendicular to each other*, both being in the wave front.

Squaring and adding equations (7), remembering that

$$(lf + mg + nh)^2 = 0,$$

it follows that

$$\alpha^2 + \beta^2 + \gamma^2 = (4\pi V)^2 (f^2 + g^2 + h^2), \text{ or}$$

$$H = 4\pi VD. \tag{8}$$

Hence,

IV. *The resultant magnetic force is $4\pi V$ times the resultant electric displacement. This is a general relation and therefore H and D are in the same phase.*

The electro-magnetic energy per unit of volume is $\frac{\mu H^2}{8\pi}$, while the electro-static energy is $\frac{KE^2}{8\pi}$. Since $D = \frac{KE}{4\pi}$,

$$H = 4\pi VD = VKE$$
$$\mu H^2 = \mu V^2 K^2 E^2. \quad \text{But } \mu K V^2 = 1$$

$$\therefore \frac{\mu H^2}{8\pi} = \frac{KE^2}{8\pi}. \tag{9}$$

Thus,

V. *At every point in the electric wave the energy is half electro-static and half electro-magnetic.*

Keeping in mind the results given by the general equations of the field for the case of a plane wave, which results have been given above for the sake of convenience, let us now assume a wave of electric force traveling along the axis of x, the force varying as the sine of an angle. The force is everywhere parallel to the yz plane, and we may put

$$Y = A \sin \frac{2\pi}{\lambda}(x - Vt). \tag{10}$$

The wave in Fig. 1 is drawn for the time $t = 0$. At O, O_1, O_2, O_3, where $x = 0, \frac{\lambda}{2}, \lambda, \frac{3}{2}\lambda$ etc., $Y = 0$. At A where $x = \frac{\lambda}{4}$, $Y = A$, its maximum value. Likewise at all points in a plane passing through A, parallel to the yz plane, the force has its maximum value A, at the instant $t = 0$; the wave is assumed to be of indefinite extent in the yz plane. Throughout the space between the yz plane and a parallel plane through A the electric force is decreasing, while between A and O, it is increasing; hence a moment later the electric force will be represented by the dotted curve, and thus the wave will have advanced a distance dx in a time dt; $\frac{dx}{dt} = V$, the velocity of the wave. The electric force Y is of course always at right angles to the direction of propagation, that is in the wave front.

The electric displacement, or polarization of the medium due to the electric force, is everywhere proportional to the force and also in the wave front, the medium being assumed isotropic. Hence if h is the displacement produced by the force Y, and K the dielectric constant,

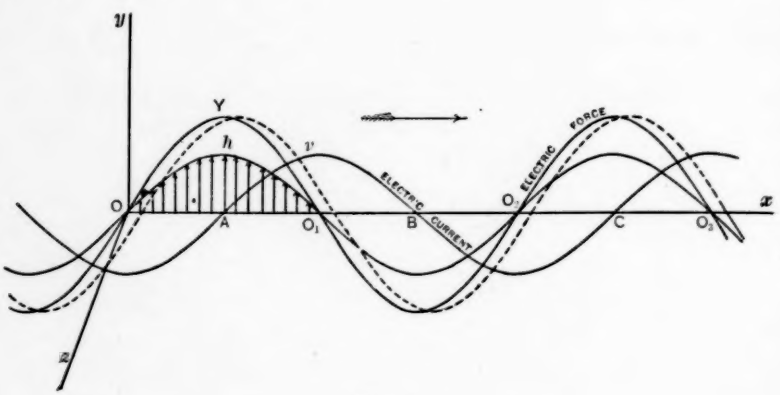


Fig. 1.

$$h = \frac{KY}{4\pi} = \frac{KA}{4\pi} \sin \frac{2\pi}{\lambda}(x - Vt). \qquad (11)$$

The curve h, in phase with the force, represents the displacement at the time $t = 0$. Throughout the entire space included between the

yz plane and a parallel plane through O, the medium is displaced upward, by an amount proportional for each stratum to the corresponding value of the ordinate of the h curve; between O_1 and O_2 the displacement is downward. Between O and A the displacement is decreasing, whereas between A and O_1, it is increasing. The wave of displacement is of course advancing with the same velocity as the force.

The displacement current, v, which is the time rate of change of the displacement, or $\frac{dh}{dt}$ is

$$v = -\frac{KA}{4\pi} \cdot \frac{2\pi V}{\lambda} \cdot \cos \frac{2\pi}{\lambda}(x - Vt).$$

$$= -\frac{KAV}{2\lambda} \cos \frac{2\pi}{\lambda}(x - Vt) \qquad (12)$$

$$= -\frac{KAV}{2\lambda} \sin \frac{2\pi}{\lambda}\left(\left(x + \frac{\lambda}{4}\right) - Vt\right). \qquad (13)$$

This is the current density, or current per square centimeter, through any plane parallel to the xz plane. Like the force Y and the displacement h, it is the same everywhere in any plane parallel to yz. It is, however, not in phase with Y and h, but as (13) shows is a quarter of a period in advance, and is represented by the v curve of Fig. 1. The currents being a maximum where the rate of change of displacement is greatest, it follows that the currents are a maximum where the force and displacement are zero, and conversely. This is the same relation that holds in the charge and discharge of a condenser.

These displacement currents, v, filling the entire space occupied by the wave, produce magnetic effects, and we must now find the magnitude and distribution of the magnetic force. To do this we recall the theorem that the work done in carrying a unit magnetic pole around any closed circuit is equal to 4π times the total current enclosed by the circuit, and is entirely independent of the distribution of current within the area of the circuit. It is also independent of the quantity or distribution of current outside the circuit. The total current will be the integral of the current density i over the area, or

$$\text{Work} = 4\pi\iint i dx dy.$$

Let the rectangle *abcd* in the plane *xz* be the path around which the unit pole is carried. If O_1 is in the middle of the rectangle, the current will be distributed symmetrically with respect to the median

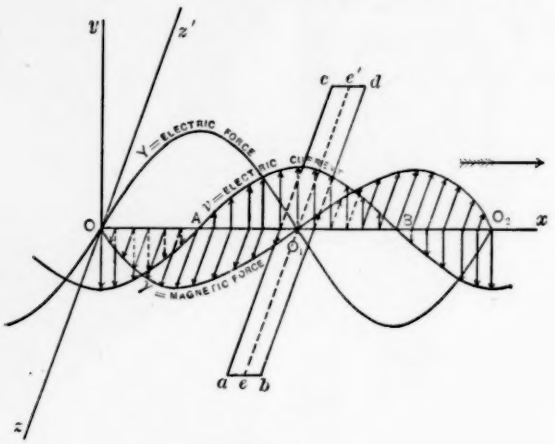


Fig. 2.

line cc'. The work done in carrying a unit magnetic pole around the rectangle is, as before, 4π times the total current enclosed, or

$$W = 4\pi\iint v dx dz.$$

If the length of the rectangle is l,

$$W = 4\pi l\int v dx. \qquad (14)$$

If the breadth of the rectangle is reduced to zero the work done is then zero. But the force being in opposite directions on the two sides of the rectangle, the word done in going around the rectangle cannot be zero unless the forces are themselves zero. Hence the magnetic force is zero along the median line cc', supposing of course there is no disturbing force due to outside influences. If the rectangle have breadth $2d$, the center of the rectangle being at O_1, the work done will be $W = 4\pi l\int_{-d}^{+d} v.dx.$

By symmetry we see that the magnetic force has the same value,

but it is in opposite direction, along the two sides of the rectangle. The work is equal to the constant magnetic force multiplied by the distance. If the wave is of infinite extent there is no force along the ends, *ab* and *cd*. Hence, calling this magnetic force γ which is parallel to the axis of z, we have

$$\gamma = -4\pi \int_0^a v dx$$

and in general

$$\gamma = -4\pi \int_0^x v dx \qquad (15)$$

(The minus sign is used here because dx to the right of O_1 is positive and γ the force is negative.) From this we see that

$$4\pi v = -\frac{d\gamma}{dx},$$

which agrees with the second of equations 1, $\frac{da}{dz}$ being zero. Thus the magnetic force would increase along the sides of the rectangle as the latter broadened out, until it reached a maximum value at A and B, being a positive maximum at A and a negative maximum at B. Then as it grew broader the rectangle would include the negative currents of the region OA and BO_2, and hence the work and also the force would decrease until at O and O_2 the force would be zero, beyond which it would increase again, becoming negative to the left of O and positive to the right of O_2. We thus readily see that the magnetic force, which is everywhere parallel to the axis of z, has its maximum and minimum values coincident with the maximum and minimum values of the electric force. In other words, the electric and magnetic force are in the same phase, are at right angles to each other, and both in the wave front.

To find the value of γ, we integrate the current v as indicated by 15. Thus,

$$\gamma = 4\pi \cdot \frac{KAV}{2\lambda} \int^x \cos \frac{2\pi}{\lambda}(x - Vt) dx$$

or

$$\gamma = KAV \sin \frac{2\pi}{\lambda}(x - Vt). \qquad (16)$$

or

$$\gamma = KVY \qquad (17)$$

Equation (16) shows that the magnetic force is a sine function and always KV times the electric force. Of course a constant of integration should be added to (16) to make it quite general. But as stated above any constant magnetic force due to outside causes produces no effect in the case here considered and may be neglected. There being no components of the magnetic and electric forces parallel to the axes of x and y and x and z respectively,

$$\gamma = H \text{ and } Y = E.$$

$$\therefore \text{ from (17)} \qquad H = KVE = 4\pi VD, \tag{18}$$

which agrees with equation (8).

But the variation of the magnetic force γ produces in turn an induced electric force which must be added to the force Y. To find the magnitude and distribution of this induced electric force we remember the theorem that *the induced electro-motive force around any closed circuit is equal to the rate of decrease of the number of lines of magnetic induction passing through it;* the relation between the positive direction of the lines of induction and the positive direction of the induced electro-motive force being given by Maxwell's corkscrew rule.

Take a narrow rectangle *abcd*, of length l and breadth dx, in the xy plane. If μ is the magnetic permeability, the number of lines of induction passing through it is $\mu\gamma l dx$. The force γ is decreasing, and the induced electro-motive force around *abcd* is

$$e = -\frac{dN}{dt} = -\mu l \frac{d\gamma}{dt} \cdot dx.$$

By the corkscrew rule it is in the direction of the arrow, *i. e.*, counter-clockwise. But the electro-motive force e is the line integral of the electric force around the rectangle, that is the work done in carrying a unit of electricity around the rectangle. Hence the electric force *up* on the right and *down* on the left is equal to

$$-\frac{\mu}{2} \cdot \frac{d\gamma}{dt} \cdot dx.$$

This is added to the electric force Y already present on the right of

the rectangle and subtracted on the left. Thus the difference due to induction will be

$$dY = -\mu \frac{d\gamma}{dt} \cdot dx \text{ or } \frac{dY}{dx} = -\mu \frac{d\gamma}{dt}. \tag{19}$$

This agrees with the third equation of (2), $\frac{dX}{dy}$ being here zero, since the force $X = 0$ everywhere. It would be the same thing to suppose a force $\frac{dY}{dx}$ acting down on the left of the rectangle in addition to Y already present and no force other than Y on the right. Then taking a series of such rectangles as in Fig. 3 we see that the in-

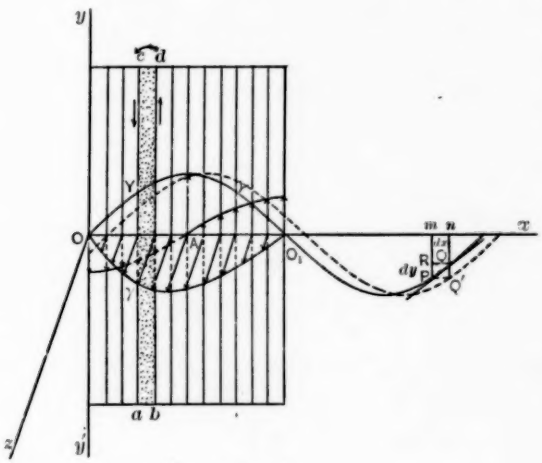


Fig. 3.

duced electric force due to the changing magnetic field is represented by the arrows drawn on the sides of the elementary rectangles, being a maximum where $\frac{d\gamma}{dt}$ is a maximum, that is where γ is zero. The magnetic field between O and A is decreasing, and between A and O is increasing; hence the electric force due to induction between O and A is downward and between A and O is upward. This tends to decrease the total force and hence the displacement between O and A and to increase them between A and O. Thus the wave is made to move forward, the reaction of the vary-

ing magnetic field cutting down the electric force on the rear slope of the wave and building it up on the front slope.

Evidently the velocity of the wave depends upon the rapidity with which this cutting down of one slope and building up of the other proceeds. At a certain instant the ordinate at a point Q is less than at P by PR, which call dY. If in the time dt the ordinate Qn is built up to $Q'n$, then the wave will be in the position of the dotted curve, and will have advanced a distance dx. Its velocity $\gamma = \frac{dx}{dt}$, is then given by the ratio of the distance dx to the time required to build up Qn to $Q'n$, at the rate $\frac{dY}{dt}$. But $QQ' = dx \tan a$, and $\tan a = -\frac{dY}{dx}$. Thus,

$$V = \frac{dx}{\frac{dx \cdot \tan a}{\frac{dY}{dt}}} = -\frac{\frac{dY}{dt}}{\frac{dY}{dx}}. \qquad (20)$$

Another way of arriving at this expression of course is to consider Y constant and find $\frac{dx}{dt}$; for the velocity of the wave is the velocity with which any constant value of Y travels forward. Thus in the above case the decrease of Y with respect to x is just counterbalanced by the increase of Y with respect to t, or the total differential of Y is zero. Hence,

$$\frac{dY}{dx}dx + \frac{dY}{dt}dt = 0, \text{ or } \frac{dx}{dt} = -\frac{\frac{dY}{dt}}{\frac{dY}{dx}}, \text{ as above.}$$

But

$$\frac{dY}{dx} = -\mu\frac{d\gamma}{dt}$$

by (19), hence,

$$V = \frac{dx}{dt} = \frac{\frac{dY}{dt}}{\mu\frac{d\gamma}{dt}} = \frac{A\frac{2\pi V}{\lambda}\cos\frac{2\pi}{\lambda}(x - Vt)}{\mu KAV\frac{2\pi V}{\lambda}\cdot\cos\frac{2\pi}{\lambda}(x - Vt)} = \frac{1}{\mu K V}$$

$$\therefore V^2 = \frac{1}{\mu K}, \text{ and } V = \frac{1}{\sqrt{\mu K}}. \qquad (21)$$

Thus, the velocity of the wave is inversely as the square root of the product of the dielectric constant and magnetic permeability, as given by (5) above.

If we square (17) and substitute $V^2 = \frac{1}{\mu K}$ we have

$$\gamma^2 = \frac{K^2 Y^2}{\mu K}, \text{ or } \frac{\mu\gamma^2}{8\pi} = \frac{KY^2}{8\pi}. \qquad (22)$$

But in this case $\gamma = H$ and $Y = E$;

$$\therefore \frac{\mu H^2}{8\pi} = \frac{KE^2}{8\pi}.$$

That is, the electro-magnetic energy for unit of volume is equal to the electro-static energy, at all places in the electro-magnetic wave, as stated in V above.

From the value of the magnetic force given in (16) we have, on substituting the value of V,

$$\gamma = \sqrt{\frac{K}{\mu}} \cdot A \sin \frac{2\pi}{\lambda}(x - Vt) \cdot = \sqrt{\frac{K}{\mu}} \cdot Y. \qquad (23)$$

That is, the magnetic force is proportional to the electric force, to the square root of the dielectric constant K, and inversely to the square root of the magnetic permeability μ. Let us see why this is so. Suppose Y and μ to remain constant, and K to be quadrupled. The magnetic force γ is then doubled, according to (23). The displacements h and the currents v will be quadrupled with K, and hence their energies will be quadrupled. But the energy of the displacement currents is due to the rotating magnetic field, and when it is quadrupled the magnetic force (represented by an angular velocity in the medium) is doubled.

Again, when μ is quadrupled, K and Y being constant, γ is only one-half its former value. The reason is that the inertia of the medium (represented by μ) being quadrupled, a certain force can set up only one-half the former velocity, the energy of rotation

being of course proportional to the square of the angular velocity. These considerations explain why in general γ is proportional to $\sqrt{\frac{K}{\mu}}$.

The velocity we found to be inversely as $\sqrt{K\mu}$. That is, increasing K or μ will decrease the velocity of the wave. First suppose K to be quadrupled, μ being constant. The displacements h and currents v will be quadrupled, as stated above, the force γ is doubled, the number of lines of magnetic induction in the field, the variation of which produces the induced electric force, is, therefore, doubled. This induced electric force $\frac{dY}{dx}$ being doubled, the wave of electric force is cut down on its rear slope and built up on its front slope twice as fast as before, as we travel along the axis of x. That is, the curve is twice as steep, and for a given period the wave-length will be only half as long, or for a given wave-length the frequency will be only half as great. Thus, in either case, the velocity is seen to be only half as great.

Again, let K be constant and μ quadrupled; γ is now only half as much as before, but $\mu\gamma$ is doubled. Hence, $\frac{dY}{dx} = -\mu\frac{d\gamma}{dt}$ is doubled and as before the velocity of the wave is reduced to one-half. Thus we see how in general the velocity of the wave is inversely as $\sqrt{K\mu}$.

We now see why the two waves (of magnetic and electric force) travel together at the same speed. Indeed, they are only the two parts of one wave. Neither could travel without the other. Suppose μ to decrease indefinitely. The magnetic energy $\frac{\mu H^2}{8\pi}$ would still be equal to $\frac{KE^2}{8\pi}$, and the magnetic force would constantly increase. The velocity of the wave would constantly increase, and when μ became zero, V would be infinite; that is, there would be no wave.

If, on the other hand, K decreased indefinitely the velocity of the wave would again increase without limit; but when $K = 0$, there would be no wave.

The energy of the medium is $\frac{\mu H^2}{8\pi} + \frac{KE^2}{8\pi}$ per cubic centimeter, the two terms representing the kinetic and potential energies respectively, and each being equal to the other, not only for the entire wave but for each element thereof. The velocity of the wave being $\frac{1}{\sqrt{K\mu}}$ we see that the amount of energy flowing with the wave past a given point per second, per square centimeter of the wave front is (since $\mu H^2 = KE^2$)

$$\frac{1}{8\pi}(\mu H^2 + KE^2)\frac{1}{\sqrt{K\mu}} = \frac{1}{4\pi}\frac{\mu H^2}{\sqrt{K\mu}}.$$

Also, $\mu H^2 = \sqrt{\mu H^2 \cdot KE^2} = \sqrt{\mu K}\ HE = \frac{1}{2}\sqrt{\mu K}\ H_1 E_1$

where H_1 and E_1 are the maximum values of H and E.

$$\therefore \quad \text{Energy per second} = \frac{H_1 E_1}{8\pi} = \frac{H_{\text{mn}} E_{\text{mn}}}{4\pi}.$$

That is, the flow of energy, which is normal to the wave front, is numerically equal to the product of the maximum magnetic and electric forces divided by 8π, or the mean values divided by 4π. This is the expression proved by Poynting to hold true generally.

It may be objected that we ought to have accounted for the origin of the wave Y, instead of assuming it as already existing. But the genesis of a wave at the beginning of a periodic force is complex, and not represented by single sine curves, as it is when once established. No claim is made for a rigorous demonstration, but rather to examine the relations of the quantities involved in the wave. We can imagine the periodic force to be due to a Hertz vibrator at the origin.

We have now followed out the relations of the various quantities concerned in a plane electro-magnetic wave, and deduced the expressions for its velocity and energy and the relative values of the electric force, displacement, current density and magnetic force.

We have found the same results as had been deduced from the general equations, and accounted for the expressions by considering the effect of a varying inertia and displacement coefficient of the medium.

To go further and attempt to account for the interior mechanism of a wave by assuming a particular structure of the ether is an interesting speculation, but that is beyond the scope of the present paper. To the professed mathematician or mathematical physicist all this may appear needless. But to the average student of physics it is such interpretations as this aims to be of the results afforded by analysis that make the subject of physics a concrete and living thing.

WESLEYAN UNIVERSITY, MIDDLETOWN, CONN., April 6, 1899.

A DETERMINATION OF THE MODULUS OF ELASTICITY FOR SMALL LOADS.

By Charles P. Weston.

AT the suggestion of Professor James S. Stevens, and under his supervision, I undertook to apply interference methods of measurement to an experimental determination of Young's modulus for copper, brass and steel. The metals were in the form of bars 66 cm. × 1 cm. × 0.5 cm. (approximate dimensions). The general method followed was the so-called "method of flexure" in which a bar, supported at each end on knife edges, is deflected by a weight applied at the center, and the deflection at the central point, together with the dimensions of the bar and the value of the deflecting force are substituted in the formula

$$M = \frac{\text{Force} \times \text{length}^3}{4 \times \text{breadth} \times \text{depth}^3 \times \text{deflection.}}$$

This formula is readily derived from a consideration of the relation between the stresses and their effects.

In my experiment the deflections were measured by the interferometer in terms of wave-lengths of sodium light and then reduced to centimeters, using 0.00005893 cm. as the value of the wave-length. The fineness of my unit of measurement allowed me to use deflecting forces very much smaller than is usual in this type of determinations.

The set up of the apparatus was as follows: The bar in test, supported by steel knife edges 60 cm. apart, was surrounded at the middle point by a loop of silk thread, A in the figure, to which the deflecting weights B could be attached. The movable mirror C and the plate to which it is attached were removed from the interferometer and the mirror alone was fastened by a pellet of soft wax to the upper side of the bar at the mid point. Then the interfe-

rometer, turned on its side, was adjusted until clear cut fringes were obtained, the source of light being a sodium flame at the right of the apparatus. At the beginning of the experiment an attempt was made to place the interferometer on a movable platform by means of which the adjustments could be made, but it was found advisable, on account of vibrations, to place it solidly on the slate table and make the adjustments by raising and lowering the knife edges or by moving the mirror on its pellet of wax for slight changes.

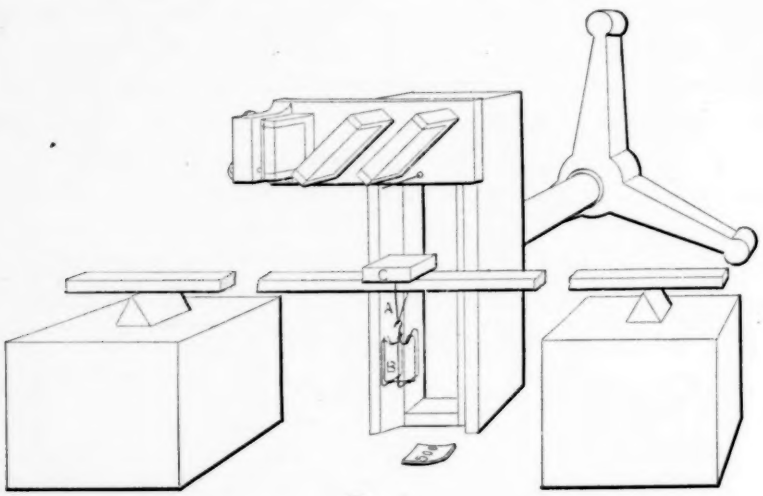


Fig. 1.

When the adjustment was satisfactory the deflecting weight was very carefully lowered until it hung suspended from the silk loop, and a count of the number of fringes passing the reference wire was made. The movable mirror was found to be so sensitive to external vibrations that no observations could be taken when any one was moving around in the building, and consequently the work had to be done in the evening or when the building was unoccupied.

For lowering the weights upon the thread slowly enough so that a good count could be made, a small platform which could be moved vertically by a tangent screw was devised. The weights used consisted of a wire basket of 0.5 g. mass and additional weights each of 0.5 g. mass, to be placed in the basket. The experiment was also tried using 0.2 g. mass and multiples, but these proved too small for reliable work.

The accompanying Tables I., II., III. show the results obtained with the three bars :

TABLE I.

STEEL BAR.

No.	Mass in g.	Defl. in Sodium waves.	Defl. in cm.	Defl. in cm. for 0.5 g = l.
1	0.5	3.0	0.00008839	0.00008839
2	1.0	6.0	0.00017678	
3	1.5	9.0	0.00026517	
4	2.0	12.0	0.00035356	

Length of bar = 60 cm. = L.

Depth of bar = 0.537 cm. = d.

Breadth of bar = 1.008 cm. = b.

Force on bar = 0.5 g. × 980.5 = F.

$$M = \frac{FL^3}{4bd^3l} = \frac{0.5 \times 980.5 \times 216000}{4 \times 1.008 \times 0.154854 \times 0.00008839}.$$

$$M = 19.19 \times 10^{11}.$$

TABLE II.

COPPER BAR.

No.	Mass in g.	Waves.	Defl. in cm. for 0.5 g. = l.
1	0.5	2.6	0.00007857
2	1.3	5.3	
3	1.5	8.0	
4	2.0		

Length = 60 cm. = L.

Breadth = 1.093 cm. = b.

Depth = 0.638 cm. = d.

$$M = \frac{FL^3}{4bd^3l} = \frac{0.5 \times 980.5 \times 216000}{4 \times 1.093 \times 0.259694 \times 0.00007857}.$$

$$M = 11.87 \times 10^{11}.$$

TABLE III.

BRASS BAR.

No.	Mass in g.	Waves.	Defl. in cm.	Defl. in cm. for 0.5 g. = l.
1	0.5	3	0.00008839	0.00008839
2	1.0	6	0.00017678	
3	1.5	9	0.00026517	
4	2.0	12	0.00035356	

Length = 60 cm. = L.

Breadth = 1.094 cm. = b.

Depth = 0.636 cm. = d.

$$M = \frac{FL^3}{4bd^3l} = \frac{0.5 \times 980.5 \times 216000}{4 \times 1.094 \times 0.257259 \times 0.00008839}.$$

$$M = 10.64 \times 10^{11}.$$

A word of explanation ought perhaps to be said in regard to the symmetry of the results in the second columns of Tables I. and III. (deflection in wave-lengths). The values were derived as follows: the deflection produced by 0.5 g. is noted several times. As nearly as can be read it is 3 waves. With the thought in mind that this is liable to be in error 0.1 either way, the deflections produced by 1.0 g., 1.5 g., etc., were noted. Any error in the first reading would become multiplied in the successive readings and be very readily detected. The fact that there is no detectable deviation from 12 complete waves for 2.0 g., argues that the first reading was a correct one. In the case of the copper bar the deflection for 1.5 g., came so exactly 8 waves that the third of that amount was used as the deflection corresponding to the excess of 0.5 g. rather than the estimated value of the deflection actually observed for 0.5 g.

After completing these determinations, using the interferometer and 0.5 g. excess, I made determinations for the same bars using excesses of 50 g. and measuring the deflections by means of a micrometer microscope reading directly to 1/28263 of a cm. The results of these determinations are embodied in the Tables IV., V., VI., while Table VII. gives a comparison of the mean values of M obtained by the different methods.

TABLE IV.

STEEL BAR.

Mass.	1st Rd.	2d Rd.	Diff.	Diff. for 50.	Mean.	l (cm.).
50	0.877	3.327	2.450	2.450		
100	0.794	5.690	4.896	2.448		
150	0.844	8.145	7.301	2.433		
200	0.828	10.633	9.805	2.451		
250	0.300	12.641	12.341	2.468	2.45	0.008668

Microscope constant = 282.63 turn to cm.
Length of bar = 60 cm. = L.
Breadth of bar = 1.008 cm. = b.
Depth of bar = 0.537 = d.

$$M = \frac{FL^2}{4bd^3l} = \frac{50 \times 980.5 \times 216000}{4 \times 1.008 \times 0.154854 \times 0.008668} = 19.56 \times 10^{11}.$$

TABLE V.

COPPER BAR.

Mass.	1st Rd.	2d Rd.	Diff.	Diff. for 50.	Mean.	l (cm.).
50 g	0.299	2.496	2.197	2.197		
100 g	0.337	4.733	4.396	2.198		
150 g	0.391	6.991	6.600	2.200		
200 g	0.435	9.200	8.765	2.191	2.1965	0.007772

$L = 60$ cm. $b = 1.093$ cm. $d = 0.638$.

$$M = \frac{FL^3}{bd^3l} = \frac{50 \times 980.5 \times 21600}{4 \times 1.093 \times 0.259694 \times 0.007772} = 12.00 \times 10.^{11}$$

TABLE VI.

BRASS BAR.

Mass	1st Rd.	2d Rd.	Diff.	Diff. for 50.	Mean.	l (cm.).
50 g	0.607	3.055	2.448	2.448		
100 g	0.040	5.070	5.030	2.515		
150 g	0.343	7.881	7.538	2.513		
200 g	0.349	10.374	10.025	2.506	2.4955	0.008868

$L = 60$ cm. $b = 1.094$ cm. $d = 0.636$ cm.

$$M = \frac{FL^3}{4bd^3l} = \frac{50 \times 980.5 \times 216000}{4 \times 1.094 \times 0.257259 \times 0.008868} = 10.616 \times 10^{11}.$$

TABLE VII.

COMPARISON.

	Interferometer.	Microm. Microp	Per Cent. of Deviation.
Steel	19.19×10^{11}	19.56×10^{11}	1.9
Copper	11.87×10^{11}	12.00×10^{11}	1.0
Brass	10.64×10^{11}	10.62×10^{11}	0.2

During the progress of the experiment an attempt was made to determine whether or not the bars returned to their original positions; that is, to determine if there was any "permanent set" for those small deflecting forces. The conclusion reached was that the return to initial position was absolute, within the limit of the fineness of observation possible with the instrument, which was to be expected, as the "set," if any, for so small a deflection would probably be beyond the range of observation even with this instrument.

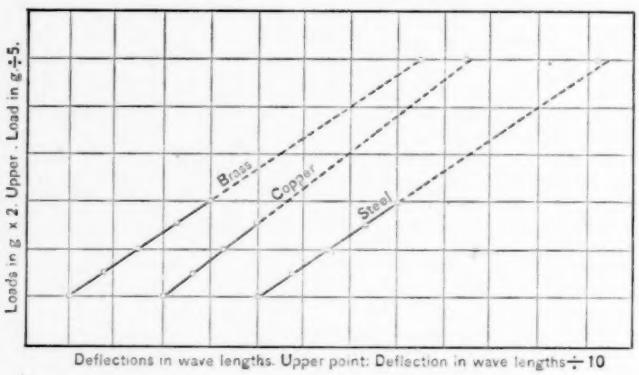


Fig. 2.

The accompanying curves show at one end the relation between loads differing by 0.5 g. and deflections measured in wave-lengths; and at the other a similar relation between loads differing by 50 g. and deflection reduced to wave-lengths, the latter reduced to a convenient scale.

DISCUSSION OF RESULTS.

Since the unit was the wave-length of sodium light it is accurately known to at least four significant figures (cm.). All measurements therefore were carried out that far; but of course the precision reached was not of that order. A movement over a dark or

yellow space is equivalent to one-quarter wave-length. These spaces could easily be estimated to tenths. The errors would, therefore, be likely to run about 1.5 to 0.4 per cent. It seems certain that in the case of copper and brass the same ratio which exists between large loads and the corresponding deflections holds for small loads and their corresponding deflections. In the case of steel the percentage of deviation between the two methods is 1.9. If we allow that an error of two per cent. might have been made in measuring the displacement of the fringes, or, what is equally probable, an error of that magnitude occurred in reading the micrometer microscope the two results will check within the limits of this error. The results in the above tables represent the mean of a series of measurements in each case. It may therefore be concluded that certainly for copper and brass and probably for steel the ratio between the deflection and a load is a constant quantity for loads ranging from 0.5 g. to those which would give the bar a permanent set.

PHYSICAL LABORATORY, UNIVERSITY OF MAINE.

A NORMAL CURVE OF MAGNETIZATION OF IRON.

BY W. S. FRANKLIN AND S. S. CLARK.

IN the ordinary method for testing iron the specimen is subjected to an impressed magnetizing force, H, which is made to pass continuously through a cycle of values. In this case the value of the induction, B, corresponding to a given value of H depends not only upon that value of H, but also upon the previous magnetic condition of the iron specimen.

It is likely that there are three distinct magnetizing forces holding each other in equilibrium in a specimen of magnetized iron, namely:

(*a*) The impressed magnetizing force including the demagnetizing force due to the free magnetic poles of the specimen.

This demagnetizing force of the specimen upon itself, due to its free magnetic poles, is equal to $\frac{dE}{dB}$ when E is the energy of the magnetic field in the region surrounding the iron due to the free poles. This energy is all essentially recoverable. The behavior of the core of the induction coil depends largely upon this store of extraneous energy, inasmuch as it is this store of energy, which in part furnishes the work needed to demagnetize the core when the circuit is interrupted, or rather this store of energy is utilized in charging the condenser the discharge of which brings about the demagnetization. It is the necessity of this store of recoverable energy which makes the closed iron magnetic circuit so ineffective with the ordinary induction coil; while with the alternating current transformer the sources of primary current furnish energy direct for both magnetizing and demagnetizing.

(*b*) A coercive or hysteretic magnetizing or demagnetizing force due to magnetic lag as the specimen is brought from one magnetic state to another. Thus, if Δh is the amount of energy per unit volume of the specimen dissipated as heat when the induction is

changed from B to $B + \Delta B$, then $-\frac{\Delta h}{\Delta B}$ is the coercive magnetizing force. The maximum values of $\frac{\Delta h}{\Delta B}$ for various values of B can be easily found by comparing the abscissæ of Ewing's cycle curve with the corresponding abscissæ of the normal curve of magnetization.

(*c*) A demagnetizing force due to a store of recoverable potential energy in the iron, or rather to a store of magnetic energy which is not yet dissipated by hysteresis. Thus, if V is the amount of this undissipated magnetic energy per unit volume, then V is a function of B and $\frac{dV}{dB}$ is the demagnetizing force due to V.

This store of recoverable energy, V, in the iron is also important in the action of the induction coil. When the core of the induction coil is worked up to very high values of B this store of recoverable internal energy becomes very considerable. This is the explanation of the fact that an ordinary induction coil with closed magnetic circuit works most satisfactorily when the core is pushed to extreme saturation. The problem involved in the design of an induction coil, aside from the interrupting mechanism, is to provide that the energy $E + V$ be more or less in excess of the energy dissipated by hysteresis upon demagnetization $\left(\int_B^0 h.dB\right)$ plus the energy which flows into the secondary coil. The quickness of demagnetization of the core upon interruption of the primary current depends in part (for ordinary speeds of demagnetization wholly) upon the smallness of the capacity of the condenser which can be used with the interrupter employed, and in part upon the remoteness of the various portions of the energy E and V from the primary and secondary coils, for these distances determine or set a limit to the quickness with which this energy can flow into the primary and secondary coils. The full discussion of the induction coil is beyond the scope of this paper, the above described actions being mentioned for the purpose of exemplifying the classification here given of the energies in an iron core and their reactions (magnetizing forces).

The function V might be determined directly as follows: The iron specimen is to be magnetized from any state for which B is equal to zero to the given induction B. During this magnetization the values of B and H are to be observed and the integral $w = \frac{1}{4\pi}\int H.dB$, which gives the work spent upon the specimen per unit volume, calculated. Further the dissipated energy, h, is to be observed, for example, by observing the rise in temperature of the specimen. The difference $w - h$ is the value of the function V for the given value of B. This method can scarcely be carried out, because of the difficulty of determinating the dissipated energy. In fact the only accurate method for measuring this dissipated energy is by using complete cycles of magnetization in which case $w = h$.

The function V may be determined indirectly by determining the curve of magnetization under conditions in which the coercive or *hysteretic* force is obliterated, for, in this case the impressed magnetizing force H and the demagnetizing force $\frac{dV}{dB}$ are at each step in equilibrium so that the curve of B and H obtained becomes the curves of B and $\frac{dV}{dB}$. This curve the authors call the *normal curve of magnetization.* This indirect method depends, of course, upon the means which are employed for obliterating the hysteretic force.

METHODS OF MEASUREMENT.

Two methods have been employed for the obliteration of the coercive force in the determination of $\frac{dV}{dB}$. In both methods the apparatus was arranged in the ordinary way for the determination of the B and H curve, Ewing's arrangement, in fact, being employed. In one method the iron specimen was subjected to the process of *demagnetization by reversals* at each step. For this purpose an alternating current was sent through an auxiliary coil, surrounding the specimen of iron and this alternating current was slowly reduced to zero before each set of readings for B and H. In the other method the iron specimen, which was in the form of a wire,

was stretched by a constant force and set into violent longitudinal vibrations before each set of readings for B and H. The curve obtained by the first method was reversible; the curve obtained by the second method was not reversible, therefore, the second method does not obliterate coercive force. The reversibility of the curve in the first case does not indicate with certainty that the coercive force was obliterated. When the operation of demagnetization by rever-

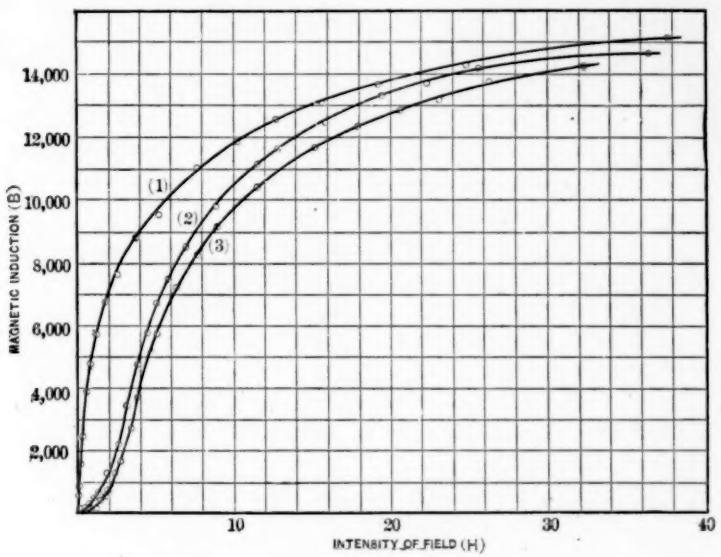


Fig. 1.

sals is performed upon a specimen of iron which is at the same time subjected to a *small* constant impressed magnetizing force, the specimen does undoubtedly settle to a state in which coercive force is zero. For *large* impressed magnetizing forces, however, the operation of demagnetization by reversals does not entirely obliterate coercive force.

Experimental Results.

Three curves of B and H were determined for a sample of annealed iron wire. Curve 1 and Table I give the results obtained when at each step the specimen was subjected to the operation of demagnetization by reversals. Curve 2 and Table II give the results obtained when at each step the specimen was subjected to vio-

lent longitudinal vibrations. Curve 3 and Table III give the results obtained when the specimen was magnetized in the ordinary way from a neutral condition. The curve Fig. 2 and Table IV give the values of the energy function V for various values of B as determined by integrating the curve 1.

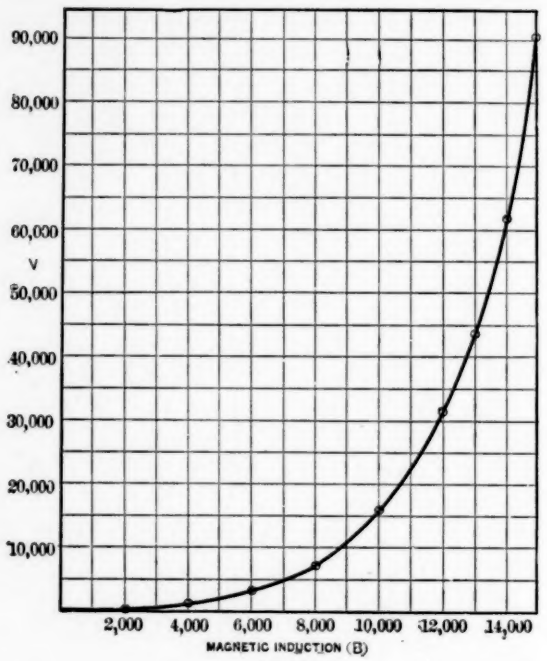


Fig. 2.

TABLE I.

Magnetizing Field (H).	Magnetic Induction (B).	Magnetizing Field (H).	Magnetic Induction (B).
0.01	116	3.85	8840
.06	608	5.23	9790
.10	965	6.40	10440
.19	1620	7.67	11000
.32	2500	10.20	11900
.64	3940	12.80	12650
.89	4770	15.60	13100
1.28	5790	19.10	13700
1.92	6870	24.00	14300
2.57	7650		

TABLE II.

Magnetizing Field (H).	Magnetic Induction (B).	Magnetizing Field (H).	Magnetic Induction (B.
0.13	29	3.90	4780
.26	116	4.52	5790
.38	184	5.18	6780
.51	251	5.82	7480
.64	319	7.03	8490
.77	367	8.95	9850
.90	445	11.60	11150
1.04	512	12.80	11590
1.16	628	16.00	12510
1.28	725	19.40	13330
1.92	1350	22.50	13800
2.57	2220	25.80	14200
3.23	3480	37.60	15190

TABLE III.

Magnetizing Field (H).	Magnetic Induction (B).	Magnetizing Field (H).	Magnetic Induction (B).
0.47	116	7.67	8260
.64	126	8.95	9170
.96	223	10.22	9870
1.28	426	11.50	10450
1.60	609	12.90	11010
1.92	870	15.30	11710
2.31	1353	18.00	12400
2.56	1672	20.70	12950
3.20	2723	23.10	13290
3.83	3824	26.30	13770
5.11	5755	32.30	14260
6.39	7260	36.50	14650

TABLE IV.

B.	V.	B.	V.
1000	90	9000	10840
2000	320	10000	15690
3000	680	11000	22420
4000	1230	12000	31550
5000	2060	13000	44110
6000	3270	14000	62280
7000	4980	15000	90110
8000	7400		

PHYSICAL LABORATORY, LEHIGH UNIVERSITY, January, 1899.

DETERMINATION OF HYSTERESIS LOSS IN IRON FOR SMALL RANGES OF INDUCTION.

BY H. S. WEBB.

THE object of this investigation was to determine the relation between hysteresis loss and induction by a method especially applicable to the measurement of the hysteresis in iron for small magnetizing forces. The hysteresis loss for various ranges of magnetizing forces was first obtained and, afterwards, the B and H curve of the specimen was determined.

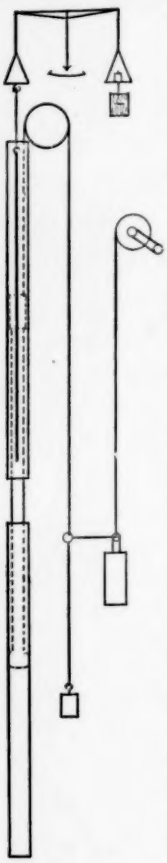


Fig. 1.

The method used for determining the hysteresis loss was devised by Prof. W. S. Franklin and described by him in a paper published in the PHYSICAL REVIEW, Vol. II., No. 12 and entitled "A New Method for Testing the Magnetic Properties of Iron." This method consists simply in weighing by a balance the force necessary to reverse the magnetism in the iron. He has shown that the energy per cubic centimeter lost in the iron in one cycle is equal to the force measured by the balance in dynes divided by area of the ironspecimen, *i. e.*, $W = \frac{F}{q}$. The general arrangement of the apparatus is shown in Figure 1. The iron specimen was 240.5 centimeters long and consisted of seven wires each .1014 centimeters in diameter. This specimen was suspended from one arm of an analytical balance, in a wooden tube 249.6 centimeters long, the hole in this tube being 2.54 cntimeteers square. The tube had two windings on it, one to neutralize the vertical component of the earth's magnetism, the other connected to the secondary of a special trans-

former arranged to give a very slowly and uniformly decreasing alternating current. This was used merely to reduce the specimen to a neutral state. Ewing's magnetometer method was used to determine when the specimen had been reduced to this neutral magnetic condition. This transformer had its secondary attached to a dash pot so that it moved slowly and very uniformly until beyond the influence of the primary.

Inside of the wooden tube was a brass tube, about 30 centimeters long, arranged by means of a long dash pot and counterpoise weights, supported by a cord running over a pulley at the top of the tube, to move uniformly and slowly up or down. It was arranged so that it required about thirty minutes to move all the way up or down. On this brass tube was a coil of 171 turns of No. 18 B. & S. copper wire, the length of the coil being 22.95 centimeters. This was the magnetizing coil by which the magnetism of the iron was reversed as the coil moved slowly up or down, the weight on the balance pan required to balance the drag of this coil being observed. These observations were taken while the coil was moving over sixty centimeters of the middle portion of the iron specimen. In order to have the magnetic quality of the specimen as uniform throughout as possible it was made of a bundle of seven wires, each annealed by an electric current.

The order of taking observations was as follows: After reducing the specimen to a neutral state, a current of a given value and direction was sent through the movable coil, call this $C+$. The coil was started down but no readings were taken. When the coil reached the bottom the current was reversed now $C-$, the coil was started up and the iron specimen was weighed as the coil moved over the sixty centimeters of the middle portion of the specimen. When the coil reached the top the current and direction of motion were again reversed and the specimen weighed as the coil moved down. The difference between these two pulls reduced to ergs per cubic centimeter of iron gave the hysteresis loss in the iron per cycle. The coil was again started up with the current in the same direction, $C+$, but no readings were taken; then the current and direction of motion were reversed and the pull observed. When the coil reached the bottom the current and direction of motion

were again reversed and the pull again observed. The order was thus as follows:

$C+$ down, no reading.
$C-$ up, balance reading.
$C+$ down, balance reading.
$C+$ up, no reading.
$C-$ down, balance reading.
$C+$ up, balance reading.

This was repeated several times for each current value in the magnetizing coil and the average pull taken. The object of this somewhat complicated procedure was mainly to eliminate apparent

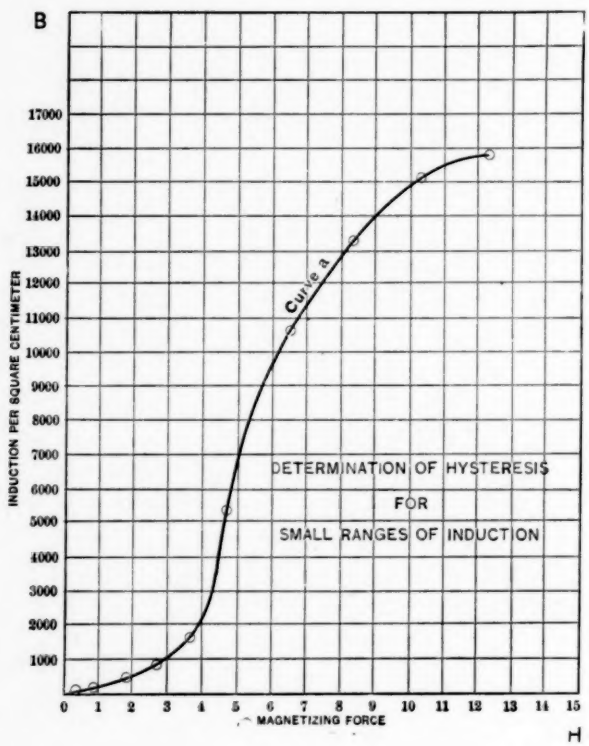


Fig. 2.

variations in the pull of the magnetized rod due to lack of uniformity in the earth's magnetic field.

The B and H curve was determined by means of the ballistic galvanometer method. The secondary coil used for determining B in the iron was wound on a paper tube just large enough in diameter to slip over the iron specimen. The following table gives the results of the observations: Curve *a* shows the relation between the magnetizing force, (H) and the induction (B). Curve *b* shows the relation between the magnetic induction (B) and the hysteresis loss in ergs per cubic centimeter per cycle (W) as determined from the observations. Curve *c* represents the most probable equation of the form $W = yB^x$ for the given set of observations in which it was found that $y = .027$ and $x = 1.45$.

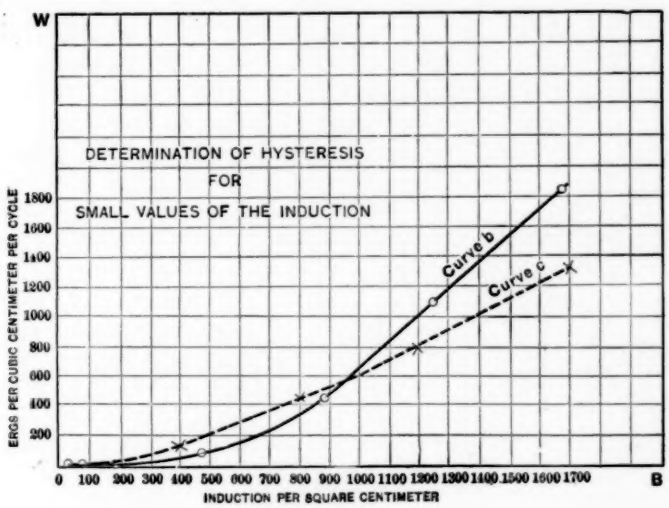


Fig. 3.

It was evident from the two curves *b* and *c* that the observations cannot be well represented by an equation of the form $W = yB^x$.

TABLE OF RESULTS.

Hysterisis Loss in Ergs per cubic centimeter per cycle.	Magnetizing Force in Moving Coil.	Induction per square centimeter (range).
W	*H*	*B*
9.89	.28	51.4
22.5	.47	80.0
31.2	.94	172.2
98.9	1.87	466.8
435.4	2.81	881.4
1090.	3.38	1250.
1818.	3.75	1672.
	4.68	5329.
	6.56	10680.
	8.43	13230.
	10.30	15110.
	12.27	15750.

At the start it was hoped that the method employed in these investigations would prove especially applicable for the measurement of hysteresis loss for small induction values. It has not, however, proved as satisfactory as was expected, for it is doubtful if the hysteresis loss can be as accurately determined by this method as by Ewing's magnetometer method, for example.

PHYSICAL LABORATORY,
LEHIGH UNIVERSITY, February, 1899.

NEW BOOKS.

A History of Physics. By FLORIAN CAJORI. 8vo, pp. 330. The Macmillan Company. 1899.

In the preface of this book is the following quotation from the announcement of Ostwald's Klassiker der Exakten Wissenschaften: "—— —— eminent and far-sighted men have repeatedly been obliged to point out a defect which too often attaches to the present scientific education of our youth. It is the absence of the historical sense and the want of knowledge of the great researches upon which the edifice of science rests."

With this thought as a key note the author might have confined himself almost entirely to "the great researches"—to the work of Newton, Faraday, Bunsen and Kirchoff, and others. But he does not do so, nor does he go to the other extreme by attempting to catalogue all who have contributed to the progress of physical science. The history is brought to the Middle Ages in twenty pages. Time is not wasted in recording the speculations, deductions and minor discoveries of the ancients, and yet enough is said to give a good idea of the spirit and learning of the time. Considering the multitude of investigations and investigators, especially of the past quarter of a century, the author has succeeded admirably in selecting and arranging his material, so well, in fact, that his special line of work is not apparent to the reader. This is more than can be said of the authors of some of our text-books on "general" physics.

In the preface is the statement that the "history is intended mainly for the use of students and teachers of physics." The word mainly might have been omitted. In the first part of the book the description of experiments and phenomena is sufficiently full to be intelligible and interesting to the general reader. But more recent developments are treated so briefly that a considerable knowledge of physics is required to read the book intelligently. For example; two lines are given to the subject of rotary magnetic field motors, five lines to the bolometer, four lines to polyphase currents, and so on. This method of treating the subject is commendable, for the book is a history, not a text-book.

It seems that the work might have been more fully illustrated. There are but eighteen figures, and most of these could have been omitted without detracting from its value. Where is the student or teacher of physics who is not familiar with Heron's "eolipile," the Magdeburg hemispheres and the Wheatstone bridge?

The last twenty pages are devoted to a history of "The Evolution of Physical Laboratories." The author traces their development from the

advent of the experimental method, and through the various stages of private laboratories, university laboratories devoted entirely to investigation, university laboratories devoted to investigation and instruction, college and secondary school laboratories for instruction, and national laboratories. There is, also, a brief discussion of the individual and class methods of laboratory instruction, the latter predominating in secondary schools and the former in universities.

A complete index, numerous foot-notes, and more than five hundred references add greatly to the value of the work. The book deserves a wide sale.

ARTHUR L. FOLEY.

Practical Electricity. By W. E. AYRTON, F.R.S. Pp. 643+xviii. Cassell & Co., Limited, London, Paris and Melbourne, 1898.

This book has been completely rewritten, and many new cuts have been introduced, while many of those found in the first edition have been omitted. The previous method of treatment of the subject has been adhered to, in which special apparatus for experimentation is described and the principles underlying its operation are elucidated. The book is written primarily for the use of students in the London Central Technical College. In that Institution all the apparatus necessary for a single experimental exercise is mounted upon a board and is connected up for immediate use upon the admission of students to the laboratory. This method of instruction has many points in its favor. A book written to be used in connection with a certain laboratory equipment, however, is of necessity limited in its usefulness. The sphere of this particular book is, however, broader than one would at first suppose, because the early date of its first appearance resulted in the equipment of many laboratories with the special apparatus described.

The explanations in some parts seem too labored for the use of American students. It seems as though the time required to be spent in experimentation with mechanical apparatus, which is sometimes quite complex, *e. g.*, that designed to prove the "law of tangents" and the "law of sines," might better be spent in learning the principles of trigonometry by the regular methods.

The selection of topics to be treated seems admirably adapted to acquaint the student with many of the facts of electrical science. Modern methods of electrical measurements of precision are, however, not touched upon or at least not more than mentioned.

SAMUEL SHELDON.

INDEX TO VOLUME VIII.

A.

Ames, J. S., *Harper's Scientific Memoirs*, 128.

Ammonia, liquid, the dielectric constant and electrical conductivity of, *Goodwin and Thompson*, 38.

Analyzer, tidal, a proposed, *Harris*, 54.

Apparatus for photographing curves of two variable currents simultaneously, *Hotchkiss*, 152.

Ayrton, W. E., *Practical Electricity*, 316.

B.

Baum, Frank G., A new transformer diagram, 95.

General method of determining the best reading of an instrument for least error in measuring a given quantity, 181.

Book Reviews

Alternate currents in practice, *Loppé and Bouquet*, 64.

American annual of photography and and *Photographic Times* almanac for 1899, 192.

Animated pictures, *Jenkins*, 63.

Barometrical determination of heights, *Cordeiro*, 255.

Elektrodynamische Grundgesetze, *Kerntler*, 61.

Elements of physics, *Gage*, 187.

Elements of physics, *Nichols ana Franklin*, 192.

Harper's scientific memoirs, *Ames*, 128.

History of physics, *Cajori*, 315.

Les modifications permanentes du verre et le déplacement du zéro des thermomètres, *Marchis*, 190.

Ostwald's Klassiker der exakten Wissenschaften, 191.

Physical geography, *Davis and Snyder*, 236.

Practical electricity, *Ayrton*, 316.

Tägliche Wärmeumsatz im Boden und die Wärmestrahlung zwischen Himmel und Erde, *Homén*, 254.

Theoretical mechanics, *Love*, 252.

Traité élémentairé de mecanique chimique, *Duhem*, 253.

Bouquet, *Alternate Currents in Practice*, 64.

C.

Cajori, Florian, *A History of Physics*, 315.

Calorimetric determination of energy dissipated in condensers, *Rosa and Smith*, 79.

Carnot engine, mechanical model of, *Shedd*, 174.

Clark, S. S., Normal curve of magnetization of iron, 304.

Comparison of thermometers, *Waidner and Mallory*, 193.

Condensers, calorimetric determination of energy dissipated in, *Rosa and Smith*, 79

Condensers, resonance method of measuring energy dissipated in, *Rosa and Smith*, 1.

Conductivity and dissociation of some electrolytes, *Foster*, 257.

Conductivity, electrical, and dielectric constant of liquid ammonia, *Goodwin and Thompson*, 38.

Conductivity of copper wire, influence of surrounding dielectric on, *Merrill*, 112.

Contact, molecular, some experiments in, *Stevens*, 49.

Convection currents produced by Röntgen rays, *Zeleny*, 161.

Cordeiro, F. J. B., *Barometrical Determination of Heights*, 255.

Currents, apparatus for photographing curves of two variable, *Hotchkiss*, 152.

Currents, method of measuring the frequency of alternating, *Kinsley*, 244.

D.

Davis, Wm. M., *Physical Geography*, 256.
Density of ice, *Nichols*, 21.
Dielectric constant and electrical conductivity of liquid ammonia, *Goodwin and Thompson*, 38.
Dielectric, influence of, on conductivity of copper wire, *Merrill*, 112.
Dissociation and conductivity of some electrolytes, *Foster*, 257.
Dissociation theory, applicability to the electrolysis of aqueous solutions containing two electrolytes with a common ion, *MacGregor*, 129.
Duhem, P., *Traité élémentairé de Mecanique Chimique*, 253.

E.

Elasticity, modulus of, for small loads, *Weston*, 297.
Electrolysis of aqueous solutions containing two electrolytes with a common ion, applicability of the dissociation theory to, *MacGregor*, 129.
Electrolytes, conductivity and dissociation of, *Foster*, 257.
Electrolytic cells, electromotive force of polarization for very small current densities, *MacNutt*, 237.
Electromotive force of polarization of electrolytic cells for very small current densities, *MacNutt*, 237.
Expansion coefficient of ice, *Nichols*, 184.
Equations, of plane electro-magnetic wave, derivations of, *Rosa*, 282.

F.

Foley, Arthur L., Cajori's *History of Physics*, 315.
Foster, William, Jr., conductivity and dissociation of some electrolytes, 257.
Franklin, W. S., Duhem's *Traité élémentairé de mecanique chimique*, 253.
Elements of Physics, 192.
Normal curve of magnetization of iron, 304.
Frequency of alternating currents, method of measuring, *Kinsley*, 244.

G.

Gage, Alfred P., *Elements of Physics*, 187.
Goodwin, H. M., On the dielectric constant and electrical conductivity of liquid ammonia, 38.

H.

Harris, Rollin A., A proposed tidal analyzer, 54.
Homén, Theodor, *Der tägliche Wärmeumsatz im Boden und die Wärmestrahlung zwischen Himmel und Erde*, 254.
Hotchkiss, H. J., Portable apparatus for photographing curves of two variable currents simultaneously, 152.
Hysteresis loss in iron for small ranges in induction, determination of, *Webb*, 310.
Hysteresis of iron and steel at ordinary temperatures and at the temperature of solid carbon dioxide, *Thiessen*, 65.

I.

Ice, coefficient of expansion of, *Nichols*, 184.
Ice, density of, *Nichols*, 21.
Iron and steel, hysteresis of, at ordinary temperatures and at the temperature of solid carbon dioxide, *Thiessen*, 65.
Iron, determination of hysteresis loss in, for small ranges of induction, *Webb*, 310.
Iron, normal curve of magnetization of, *Franklin and Clark*, 304.

J.

Jenkins, C. F., *Animated Pictures*, 63.

K.

Kann, Leopold, Radiation phenomena of the Balmain luminous paint, 250.
Kerntler, Franz, *Die electrodynamischen Grundgesetze und das eigentliche Elementargesetz*, 61.

Kinsley, Carl, Method of measuring the frequency of alternating currents, 244.

L.

Least error in measuring a given quantity, general method of determining the best reading of an instrument for, *Baum*, 181.

Logarithmic decrement and period of a continuously vibrating wire, *Thompson*, 141.

Loppé, *Alternating Currents in Practice*, 64.

Love, A. E. H., *Theoretical Mechanics*, 252.

Luminous paint, radiation phenomena of, *Kann*, 250.

M.

MacGregor, J. G., The applicability of the dissociation theory to the electrolysis of aqueous solutions containing two electrolytes having a common ion, 129.

MacNutt, Barry, Electromotive force of polarization of electrolytic cells for very small current densities, 237.

Magnetization of iron, normal curve of, *Franklin and Clark*, 304

Mallory, Francis, Comparison of thermometers, 193.

Marchis, L., *Les Modifications Permanentes du Verre et le Déplacement du Zéro des Thermomètres*, 190.

Merrill, Joseph Francis, Influence of the surrounding dielectric on the conductivity of copper wire, 112.

Merritt, Ernest, Kerntler's *Die elektrodynamischen Grundgesetze und das eigentliche Elementargesetz*, 61.

Modulus of elasticity for small loads, determination of, *Weston*, 297.

Molecular contact, some experiments in, *Stevens*, 49.

N.

Nichols, E. L., Ames' *Harpers' Scientific Memoirs*, 128.

Cordeiro's *Barometrical Determination of Heights*, 255.

Elements of Physics, 192.

Homén's *Wärmeumsatz im Boden*, 254.

Jenkins' *Animated Pictures*, 63.

Marchis' *Les Modifications Permanentes du Verre et le Déplacement du Zéro des Thermomètres*, 190.

Note on the coefficient of expansion of ice, 184.

On the density of ice, 21.

Ostwald's Klassiker der Exalten Wissenshaften, 191.

P.

Period and logarithmic decrement of a continuously vibrating wire, *Thompson*, 141.

Photographing curves of two variable currents simultaneously, apparatus for, *Hotchkiss*, 152.

Polarization of electrolytic cells for very small current densities, electromotive force of, *MacNutt*, 237.

Potential gradient of the electrodes in conduction produced by Röntgen rays, *Zeleny*, 161.

R.

Radiation phenomena of the Balmain luminous paint, *Kann*, 250.

Resonance method of measuring energy dissipated in condensers, *Rosa and Smith*, 1.

Röntgen rays, convection currents and fall of potential at the electrodes produced by, *Zeleny*, 161.

Rosa, Edward B., A resonance method of measuring energy dissipated in condensers, 1.

Calorimetric determination of energy dissipated in condensers, 79.

Derivations of the equations of a plane electro-magnetic wave, 282.

S.

Sharp, C. H., Gage's *Elements of Physics*, 187.

Shedd, John C., A mechanical model of the Carnot engine, 174.

Sheldon, Samuel, Ayrton's *Practical Electricity*, 316.

Solutions, aqueous, containing two electrolytes with a common ion, applicability of dissociation theory to the electrolysis of, *MacGregor*, 129.

Smith, Arthur W., A resonance method of measuring energy dissipated in condensers, 1.

Calorimetric determination of energy dissipated in condensers, 79.

Snyder, Wm. H., *Physical Geography*, 256.

Stevens, James S., Some experiments in molecular contact, 49.

T.

Thermometers, comparison of, *Waidner and Mallory*, 193.

Thiessen, A. H., Hysteresis of iron and steel at ordinary temperatures and at the temperature of solid carbon dioxide, 65.

Thompson, Joseph O., Period and logarithmic decrement of a continuously vibrating wire, 141.

Thompson, Maurice de Kay, Jr., On the dielectric constant and electrical conductivity of liquid ammonia, 38.

Tidal analyzer, a proposed, *Harris*, 54.

Transformer diagram, a new, *Baum*, 95

V.

Vibrating wire, period and logarithmic decrement of, *Thompson*, 141.

W.

Waidner, Charles W., Comparison of thermometers, 193.

Wave, plane electro-magnetic, derivation of equations of, *Rosa*, 282.

Webb, H. S., Determination of hysteresis loss in iron for small ranges in induction, 310.

Weston, Charles P., Determination of the modulus of electricity for small loads, 297.

Wright, T. W., Love's *Theoretical Mechanics*, 252.

Z.

Zeleny, John, On convection currents and on the fall of potential at the electrodes in conduction produced by Röntgen rays, 161.

Erratum: page 136 ninth line from bottom; for *six* read *three*.